Termination Resilience Static Analysis

Seminari di Informatica, Università di Parma

Caterina Urban
Inria & École Normale Supérieure | Université PSL

Which Non-Termination Alarm is Worse?

```
function f(x) {

  [1]...
  [2]z ← 10
  [3]if ( ... ) then
      while [4](z ≥ 0) do          ← diverges when x = 0
        [5]z ← z - x
      od[6]
  else
      while [7](z ≥ x) do          ← diverges when c ≥ 0
        [8]c ←[-2, 1]              ← non-deterministic value choice
        [9]z ←z + c
      od[10]
  fi

}[11]
```

Which Non-Termination Alarm is Worse?

Robust Non-Termination

```
function f(x) {

  ¹...
  ²z ← 10
  ³if ( ... ) then
     while ⁴(z ≥ 0) do
       ⁵z ← z - x
     od⁶
  else
     while ⁷(z ≥ x) do
       ⁸c ←[-2, 1]
       ⁹z ←z + c
     od¹⁰
  fi

}¹¹
```

← diverges when $x = 0$ (loop at 4)

← diverges when $c \geq 0$ (loop at 7)

← non-deterministic value choice (line 8)

Robust Non-Termination

∃ Input ∀ Non-Deterministic Choices : Program Diverges

```
function f(x){                    ← demonic non-determinism

  1...
  2z ← 10
  3if ( ... ) then
     while 4(z ≥ 0) do            ← diverges when x = 0
       5z ← z - x
     od6
  else
     while 7(z ≥ x) do
       8c ←[-2, 1]
       9z ←z + c
     od10
  fi

}11
```

Termination Resilience

∀ Inputs ∃ Non-Deterministic Choice : Program Terminates

```
function f(x) {

  1...
  2z ← 10
  3if ( ... ) then
      while 4(z ≥ 0) do
        5z ← z - x
      od6
  else
      while 7(z ≥ x) do
        8c ←[-2, 1]
        9z ←z + c
      od10
  fi

}11
```

👌 ← terminates when **c < 0**, **independently of the value of x** (annotation on line 7: while (z ≥ x) do)

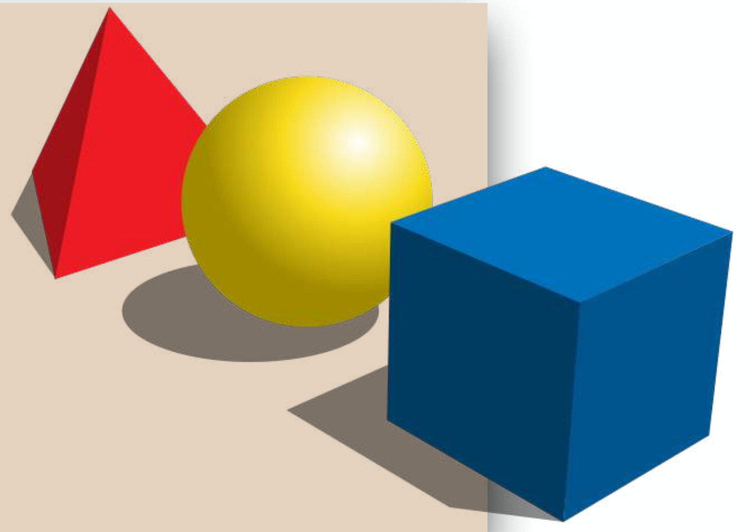
← angelic non-determinism (annotation on line 8: c ←[-2, 1])

Termination Resilience Static Analysis

3-Step Recipe

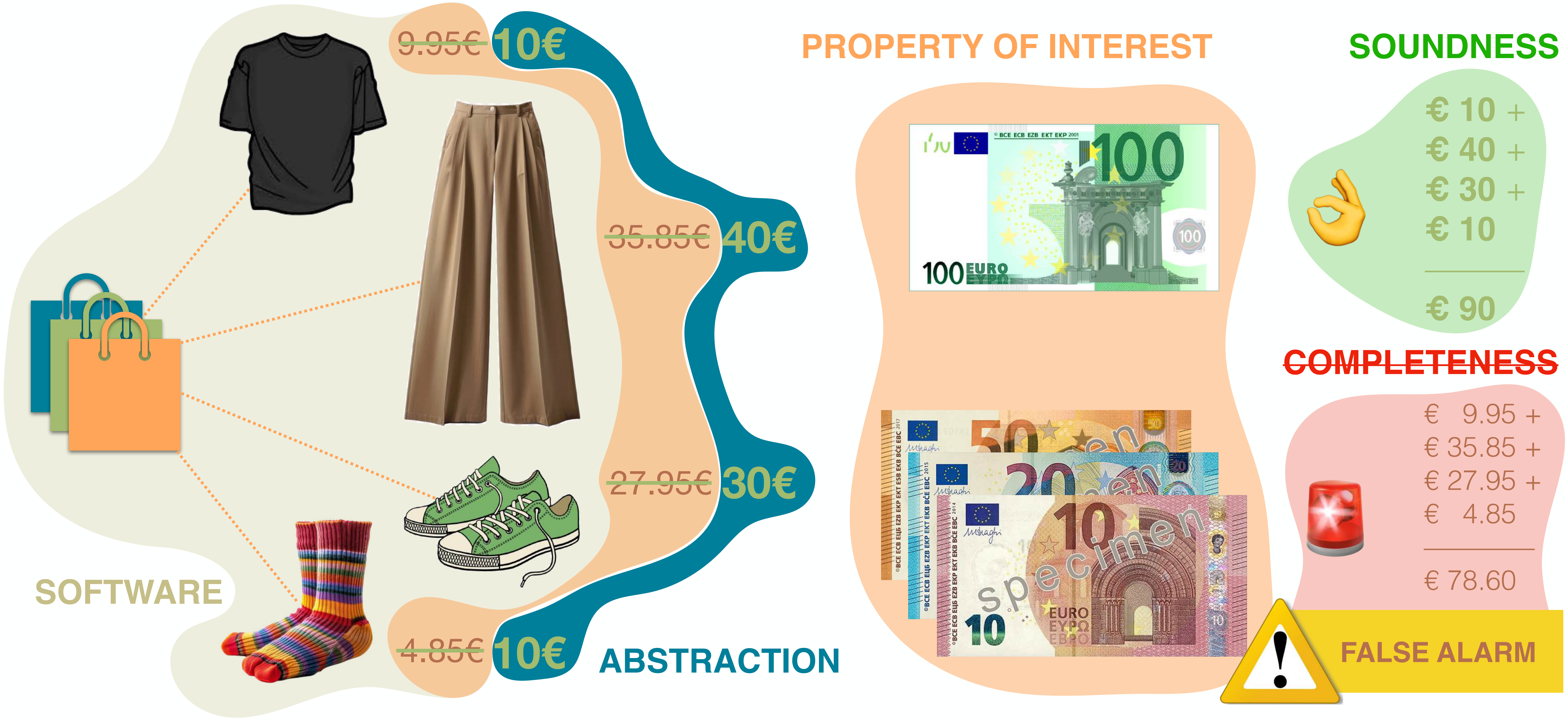
practical tools
targeting specific programs

abstract semantics, abstract domains
algorithmic approaches to decide program properties



concrete semantics
mathematical models of the program behavior

Static Analysis by Abstract Interpretation



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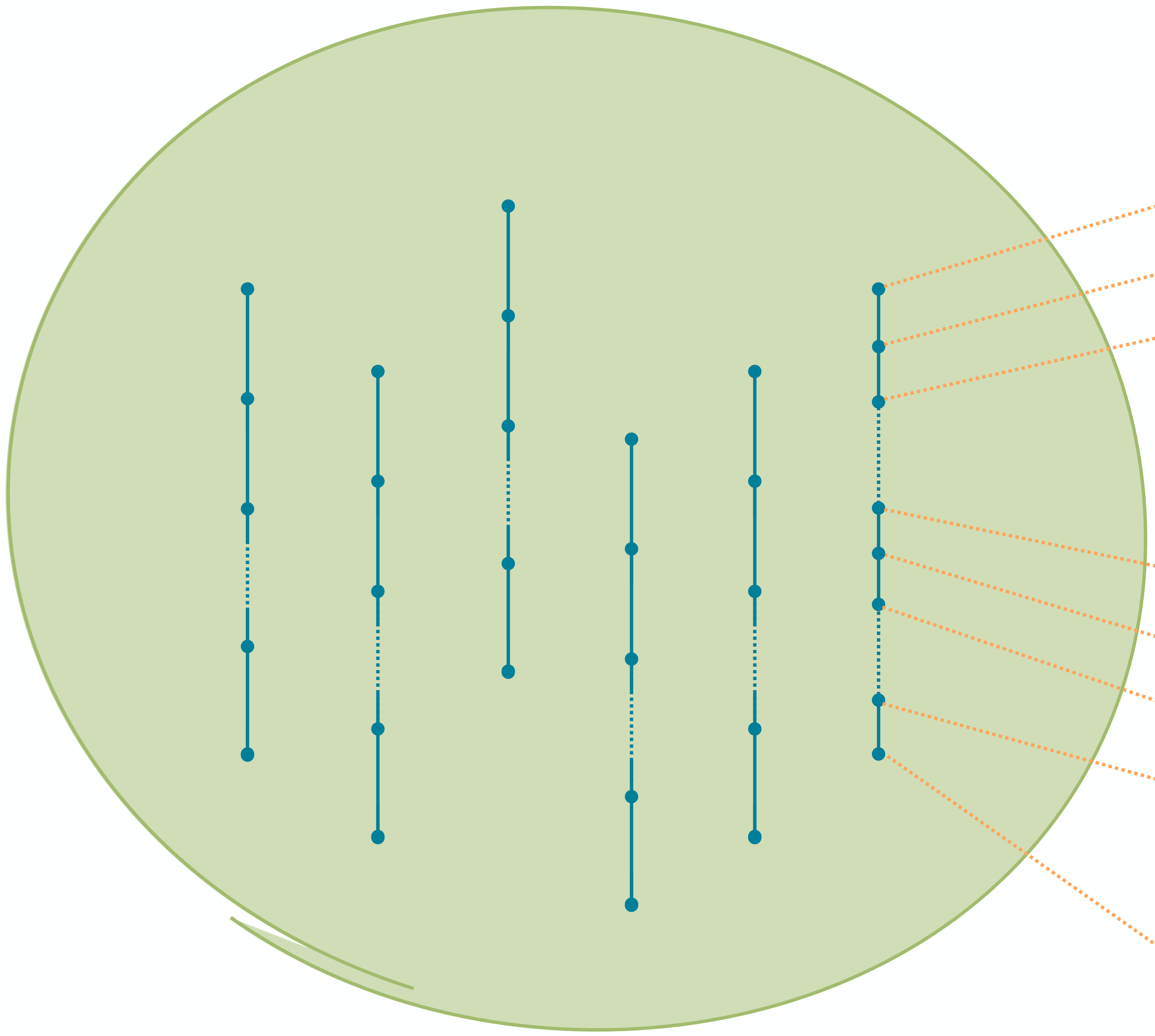
Trace Semantics




```
function f(x) {
  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
      while 4(z ≥ 0) do
        5z ← z - x
      od6
  else
      while 7(z ≥ x) do
        8c ←[-2, 1]
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      od10
  fi
}11
```
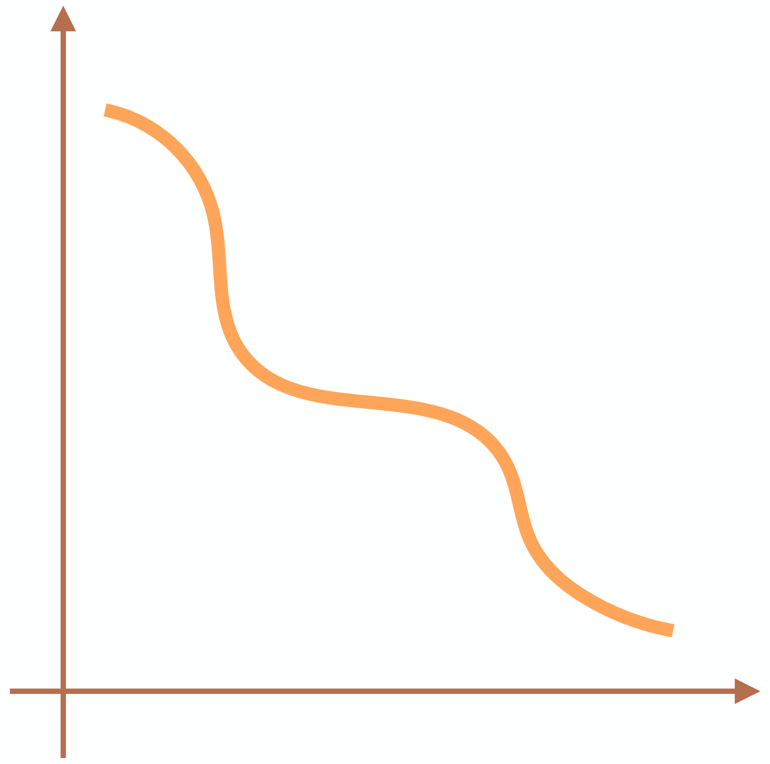
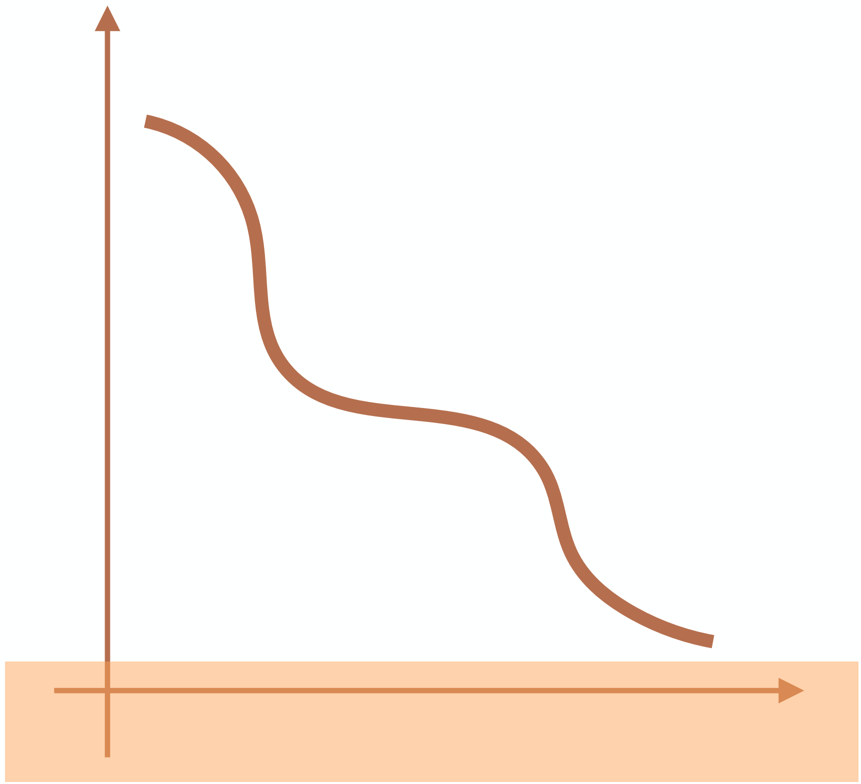

Ranking Functions

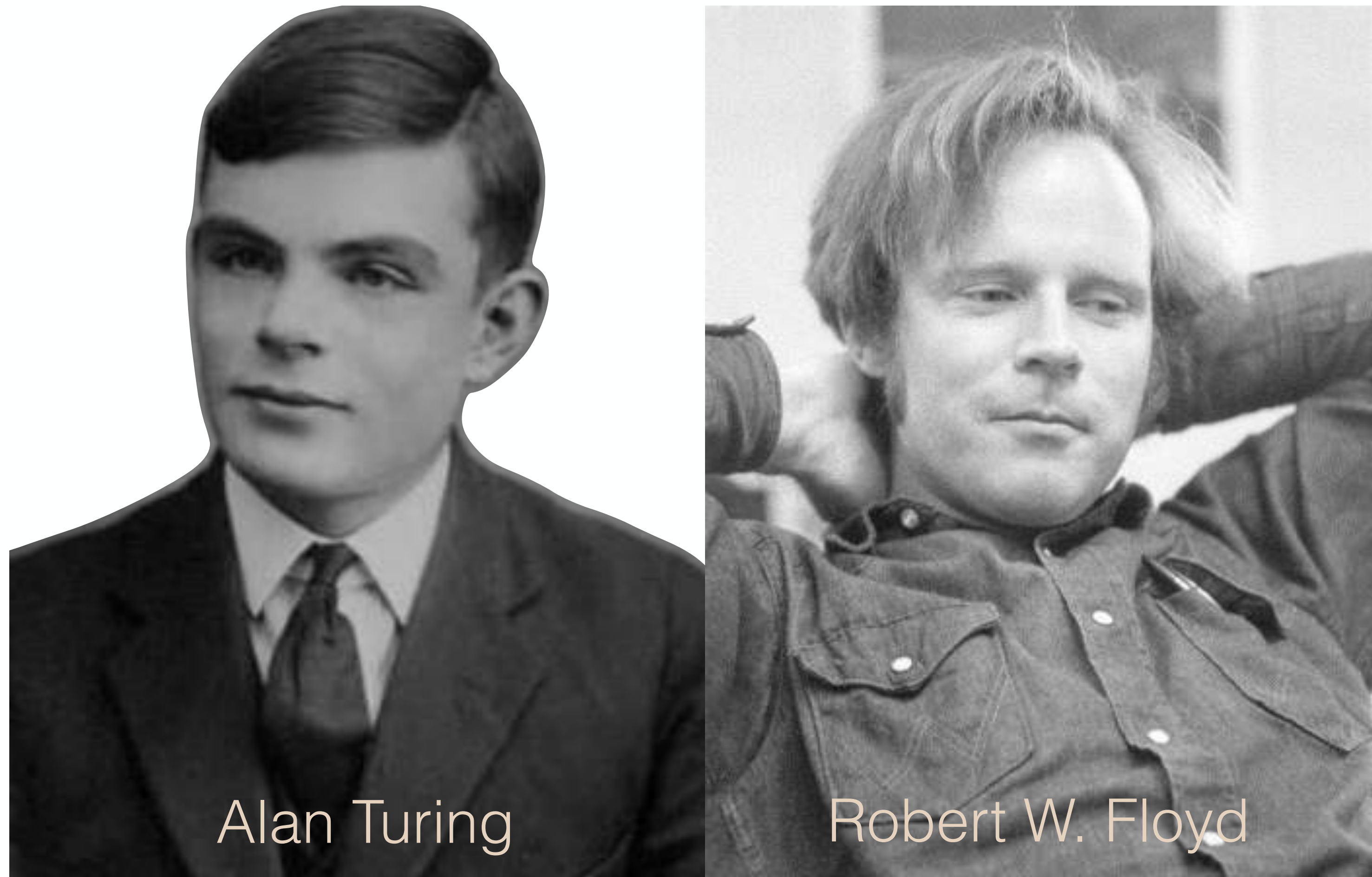
Traditional Method for Proving Termination

strictly decreasing along the execution of a program…

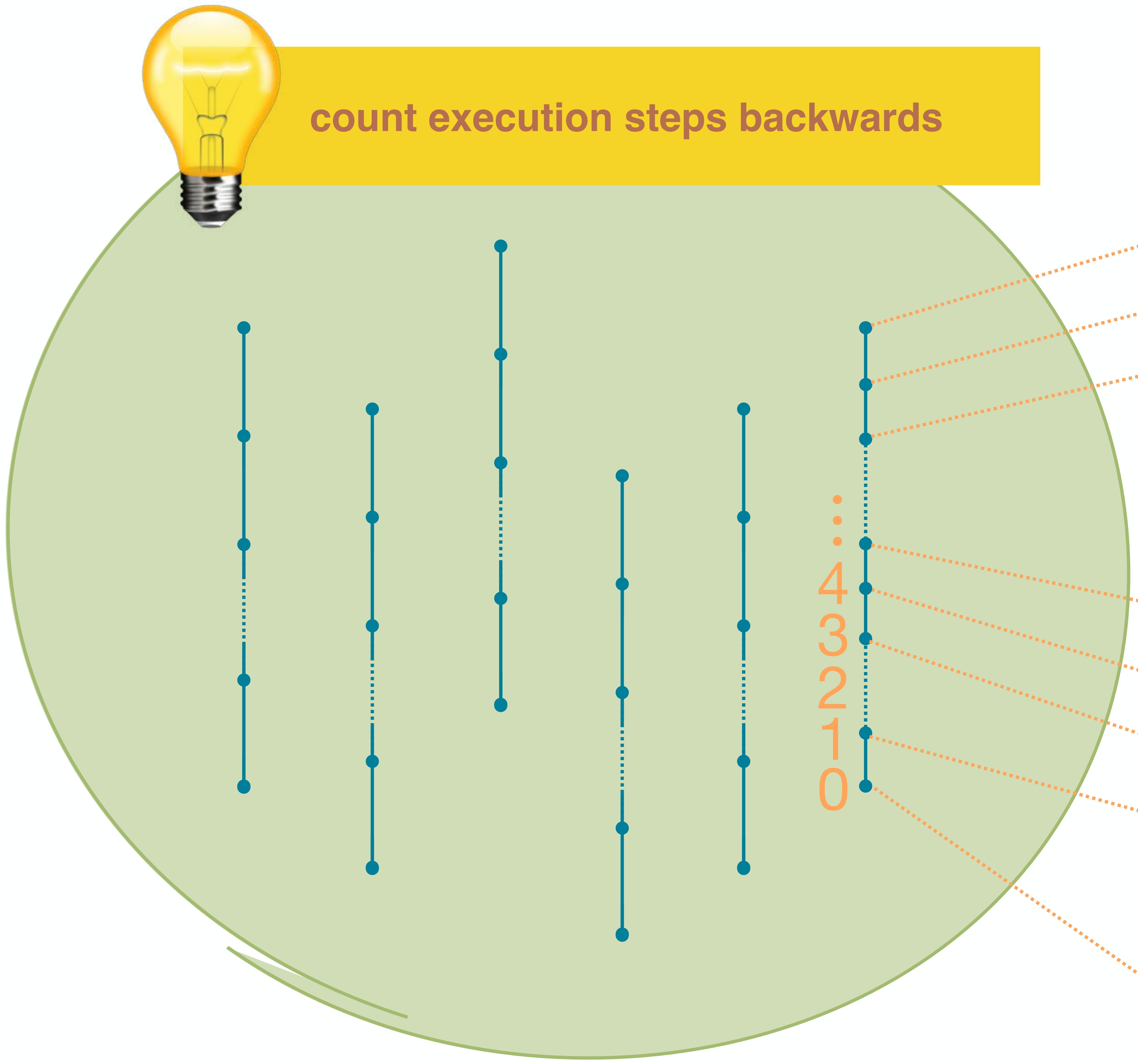


…and **well-founded**





Termination Resilience Semantics




```
function f(x) {

  ¹a ← [-∞, +∞]
  ²z ← 10
  ³if (a*a ≥ 0) then
      while ⁴(z ≥ 0) do
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  else
      while ⁷(z ≥ x) do
        ⁸c ←[-2, 1]
        ⁹z ←z + c
      od¹⁰
  fi

}¹¹
```

Termination Resilience Semantics

$$f_1 \sqsubseteq f_2 \overset{\text{def}}{=} \text{dom}(f_1) \subseteq \text{dom}(f_2) \wedge \forall x \in \text{dom}(f_1) \colon f_1(x) \leq f_2(x)$$

$$\Theta \overset{\text{def}}{=} \text{lfp}^{\sqsubseteq}_{\dot\emptyset}\ \lambda f\, \lambda s\, . \begin{cases} 0 & s \in \Omega_\tau \\ \sup\{f(s') + 1 \mid \langle s, s' \rangle \in \tau\} & s \in \widetilde{\text{pre}}_{\tau^{\text{i}}}(\text{dom}(f)) \\ \inf\{f(s') + 1 \mid \langle s, s' \rangle \in \tau\} & s \in \text{pre}_{\tau^{\text{r}}}(\text{dom}(f)) \\ \text{undefined} & \text{otherwise} \end{cases}$$

final states: $s \in \Omega_\tau$

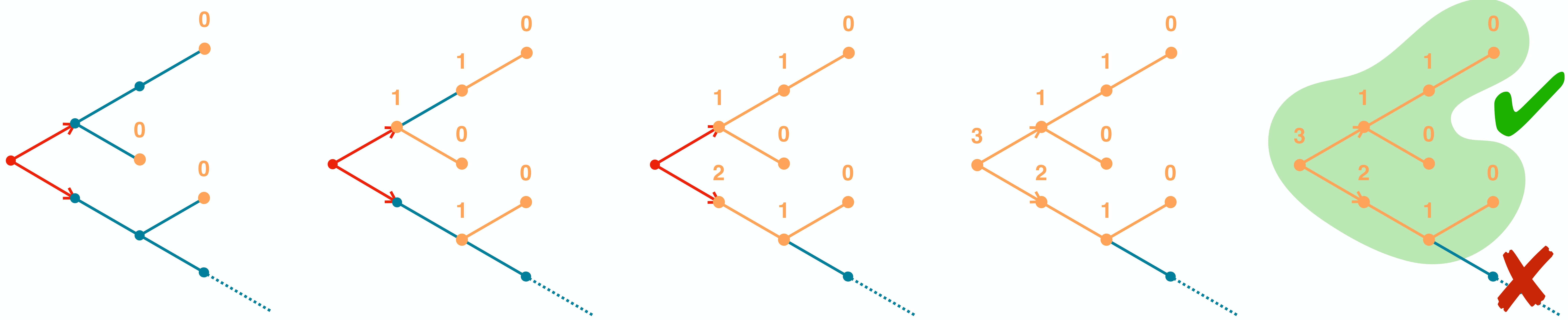
$$\widetilde{\text{pre}}_{\tau^{\text{i}}}(X) \overset{\text{def}}{=} \{s \mid \forall s' \colon \langle s, s' \rangle \in \tau^{\text{i}} \Rightarrow s' \in X\}$$

input transitions: τ^{i}

regular transitions: τ^{r}

$$\text{pre}_{\tau^{\text{r}}}(X) \overset{\text{def}}{=} \{s \mid \exists s' \in X \colon \langle s, s' \rangle \in \tau^{\text{r}}\}$$

totally undefined function: $\dot\emptyset$



Termination Resilience Semantics

$$\Theta \stackrel{\text{def}}{=} \text{lfp}^{\sqsubseteq}_{\emptyset}\ \lambda f\, \lambda s\,.\begin{cases} 0 & s \in \Omega_\tau \\ \text{sup}\{f(s') + 1 \mid \langle s, s' \rangle \in \tau\} & s \in \widetilde{\text{pre}}_{\tau^{\text{i}}}(\text{dom}(f)) \\ \text{sup}\{f(s') + 1 \mid \langle s, s' \rangle \in \tau\} & s \in \text{pre}_{\tau^{\text{r}}}(\text{dom}(f)) \\ \text{undefined} & \text{otherwise} \end{cases}$$

the existence of the fixpoint is not guaranteed

```
1x ← [-∞, +∞]
while 2(x ≠ 0) do
    3x ← [-∞, +∞]
od4
```

$$\lambda x\,.\begin{cases} 1 & x = 0 \\ \text{undefined} & \text{otherwise} \end{cases} \qquad \lambda x\,.\begin{cases} 3 & x = 0 \\ \text{undefined} & \text{otherwise} \end{cases} \qquad \dots$$

$\lambda x.0$

Termination Resilience Static Analysis

3-Step Recipe

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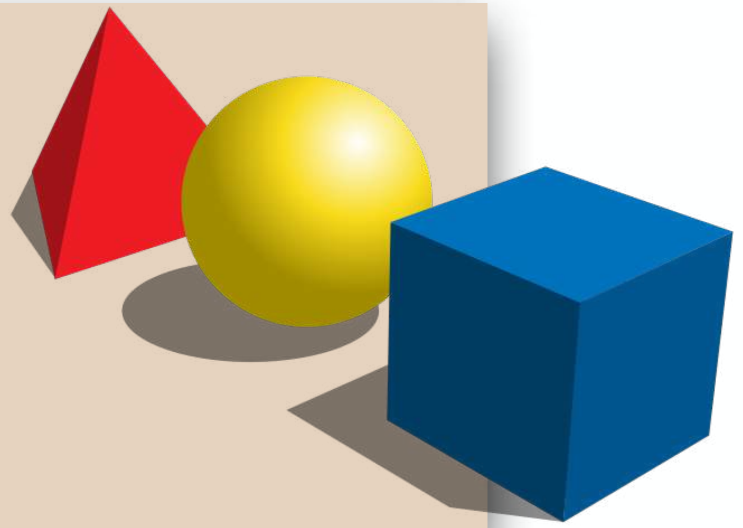
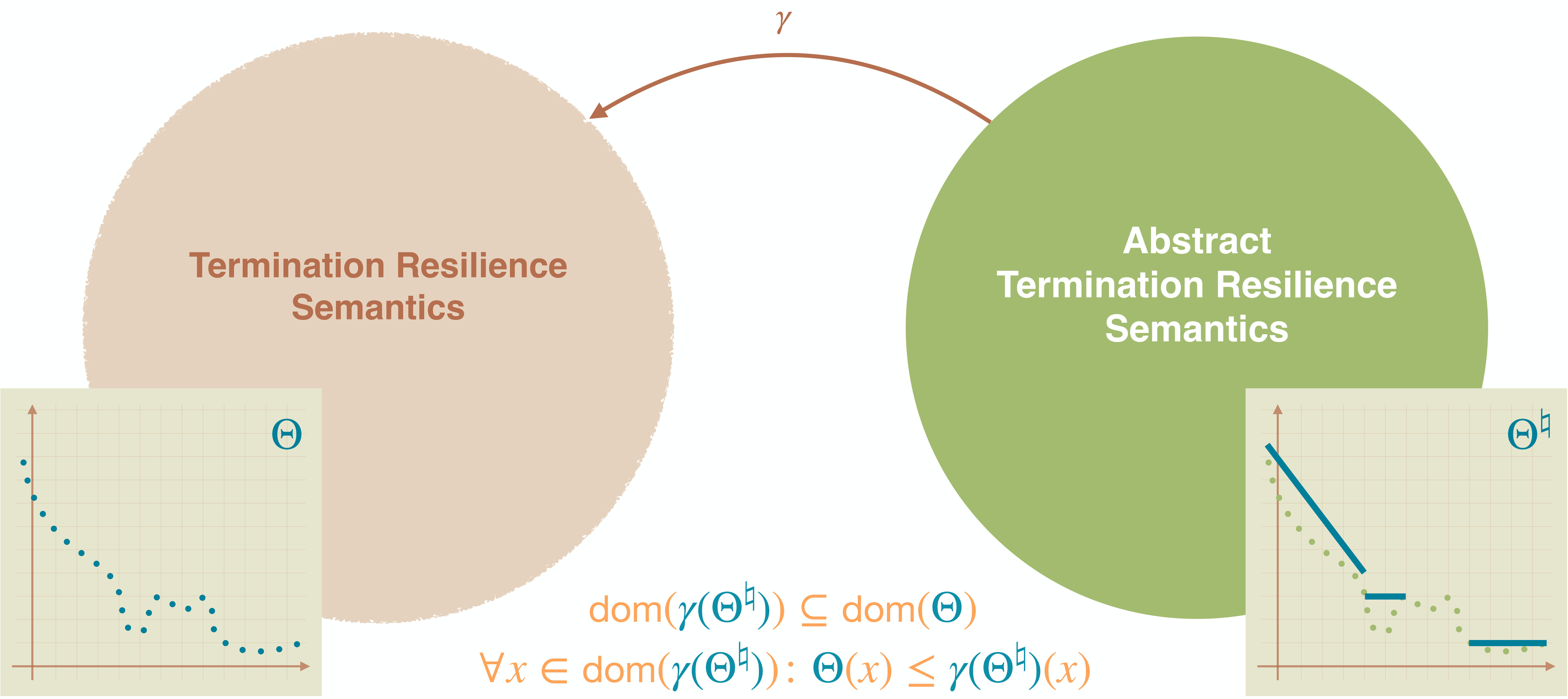


concrete semantics
mathematical models of the program behavior

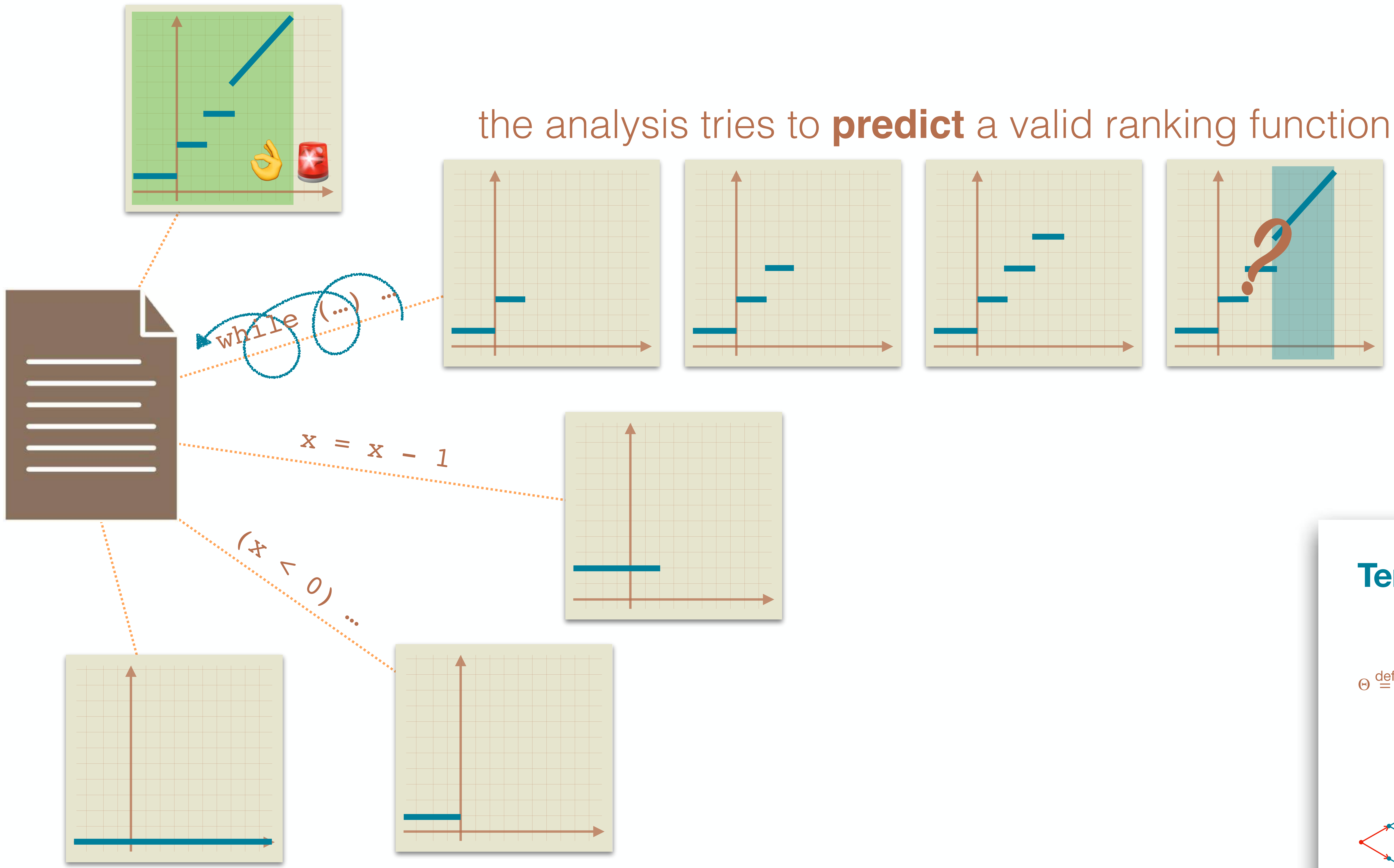
Piecewise-Defined Ranking Functions

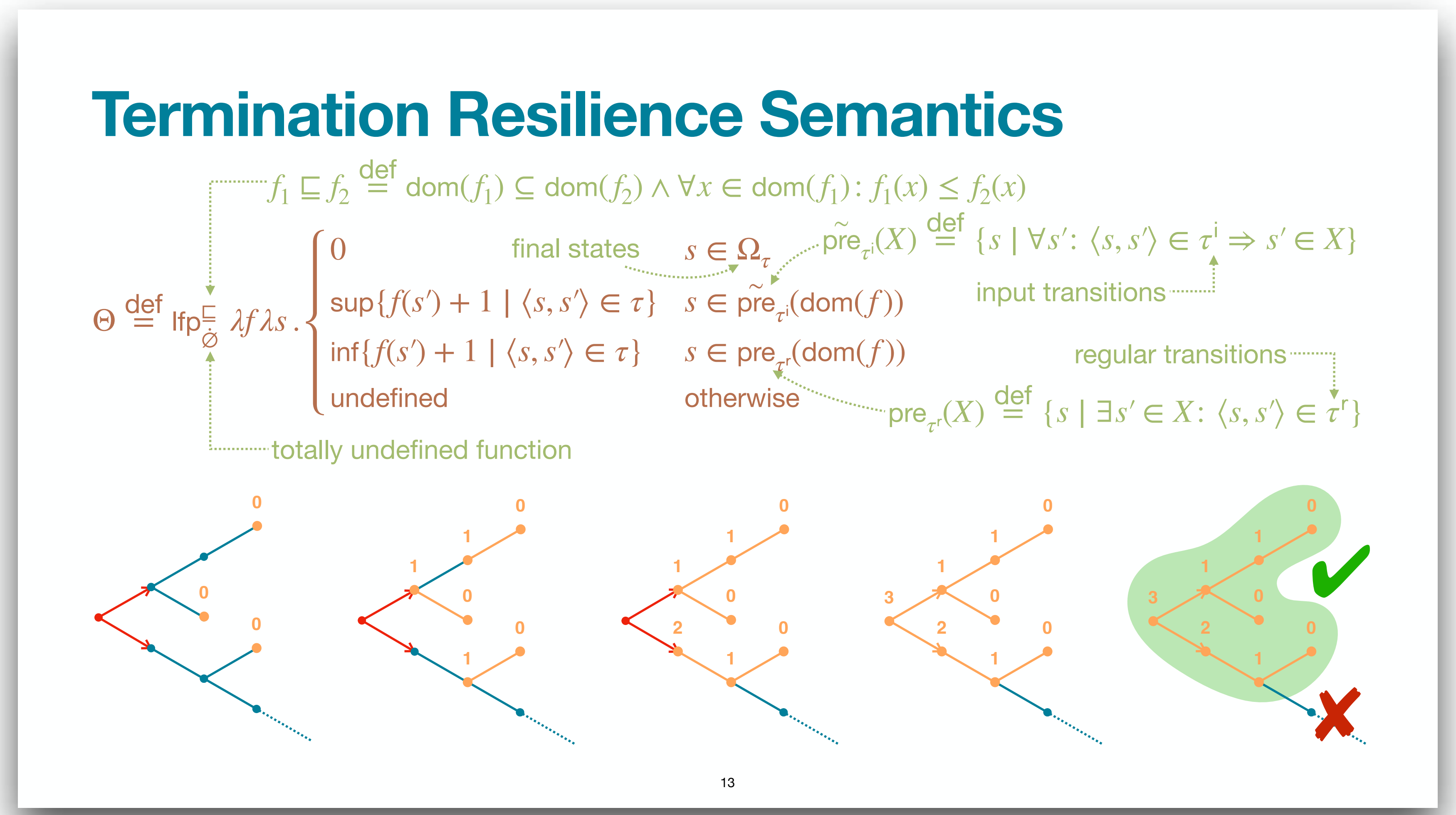


$$\text{dom}(\gamma(\Theta^\natural)) \subseteq \text{dom}(\Theta)$$

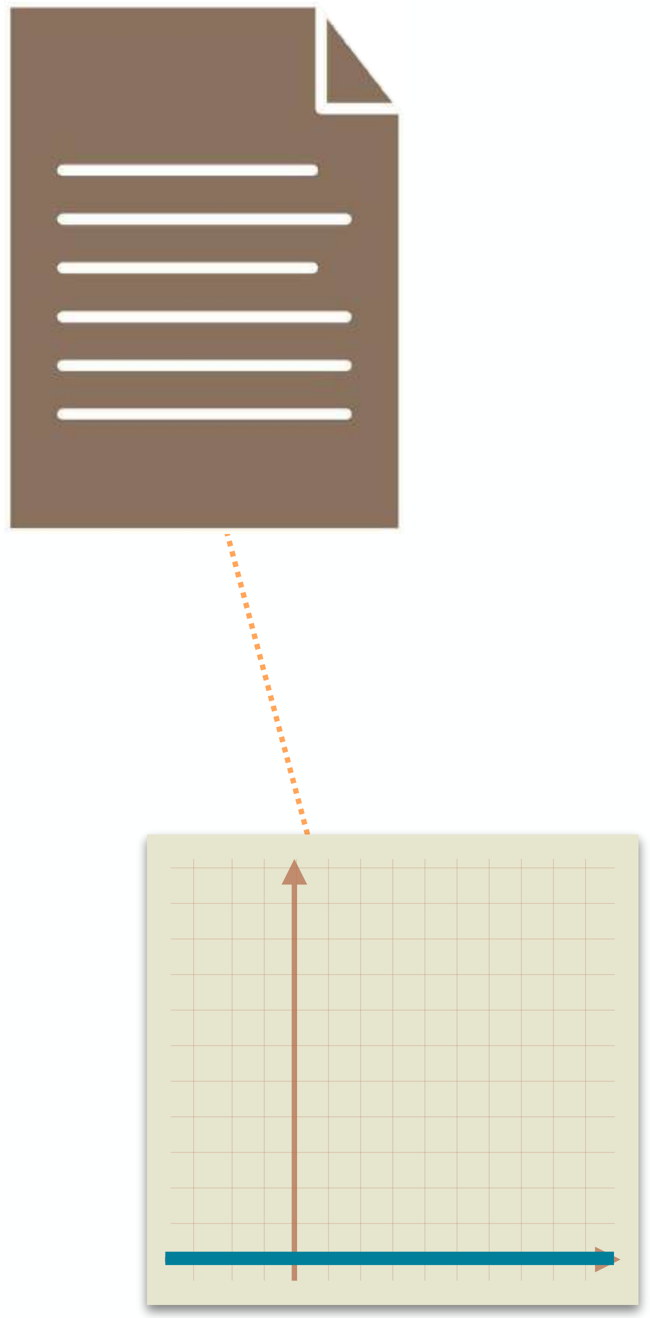
$$\forall x \in \text{dom}(\gamma(\Theta^\natural)) : \Theta(x) \leq \gamma(\Theta^\natural)(x)$$

Termination Resilience Static Analysis





Termination Resilience Static Analysis

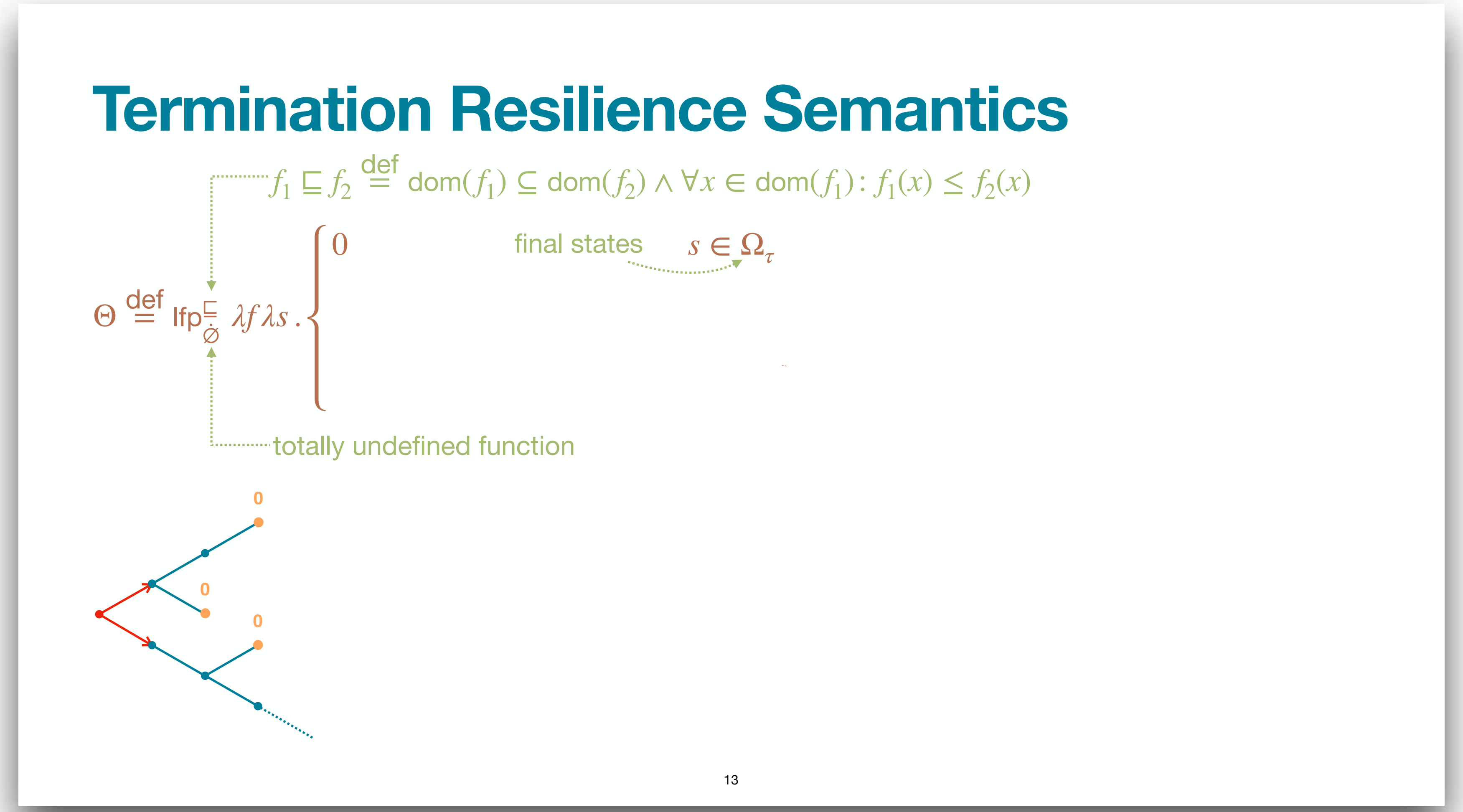


Termination Resilience Static Analysis

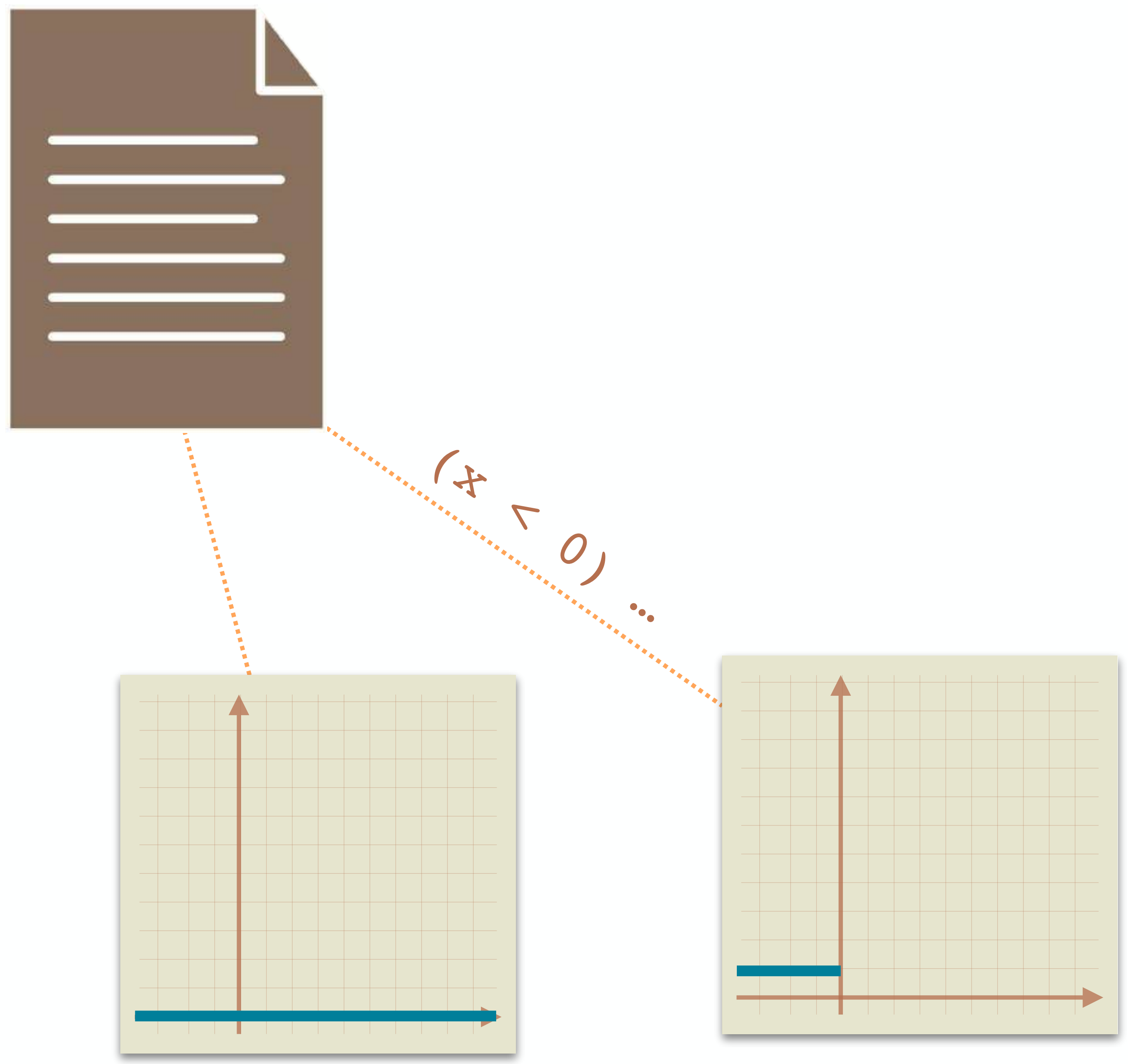
Static Backward Analysis

```
function f(x) {

  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
     while 4(z ≥ 0) do
       5z ← z - x
     od6
  else
     while 7(z ≥ x) do
       8c ←[-2, 1]
       9z ←z + c
     od10
  fi

}11
```

$\lambda xzac.\ 0$



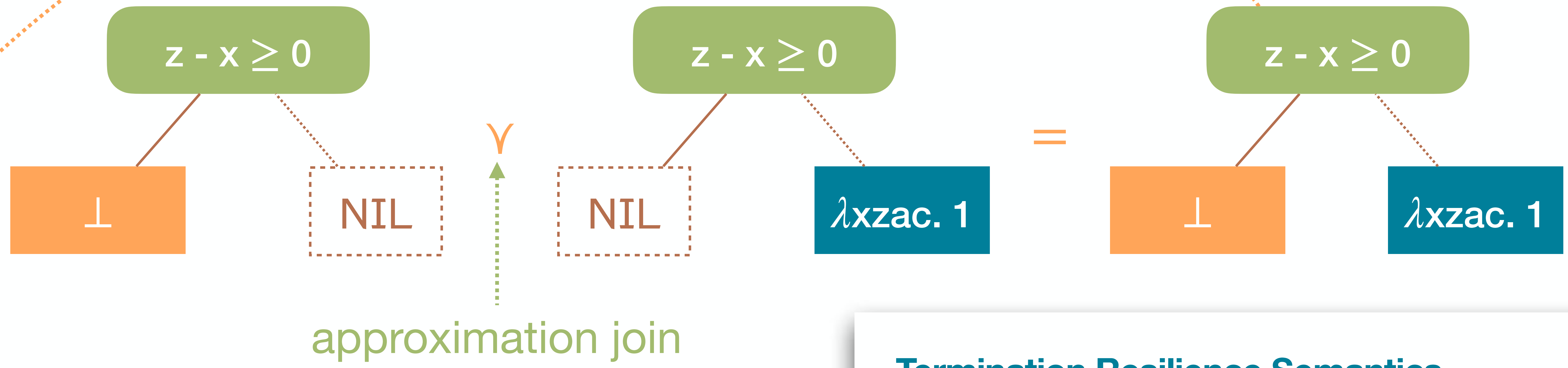
Termination Resilience Static Analysis

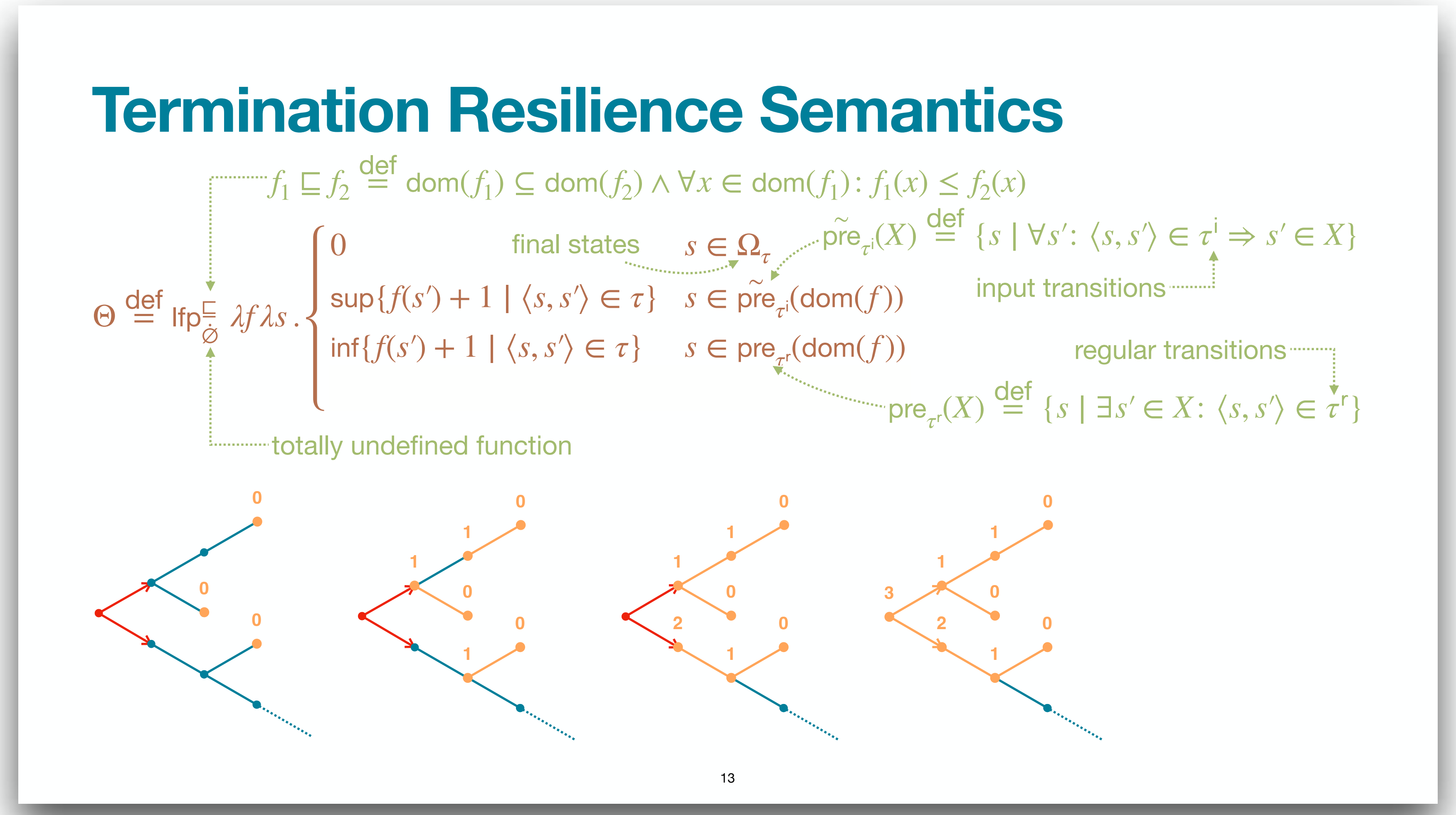


Termination Resilience Static Analysis

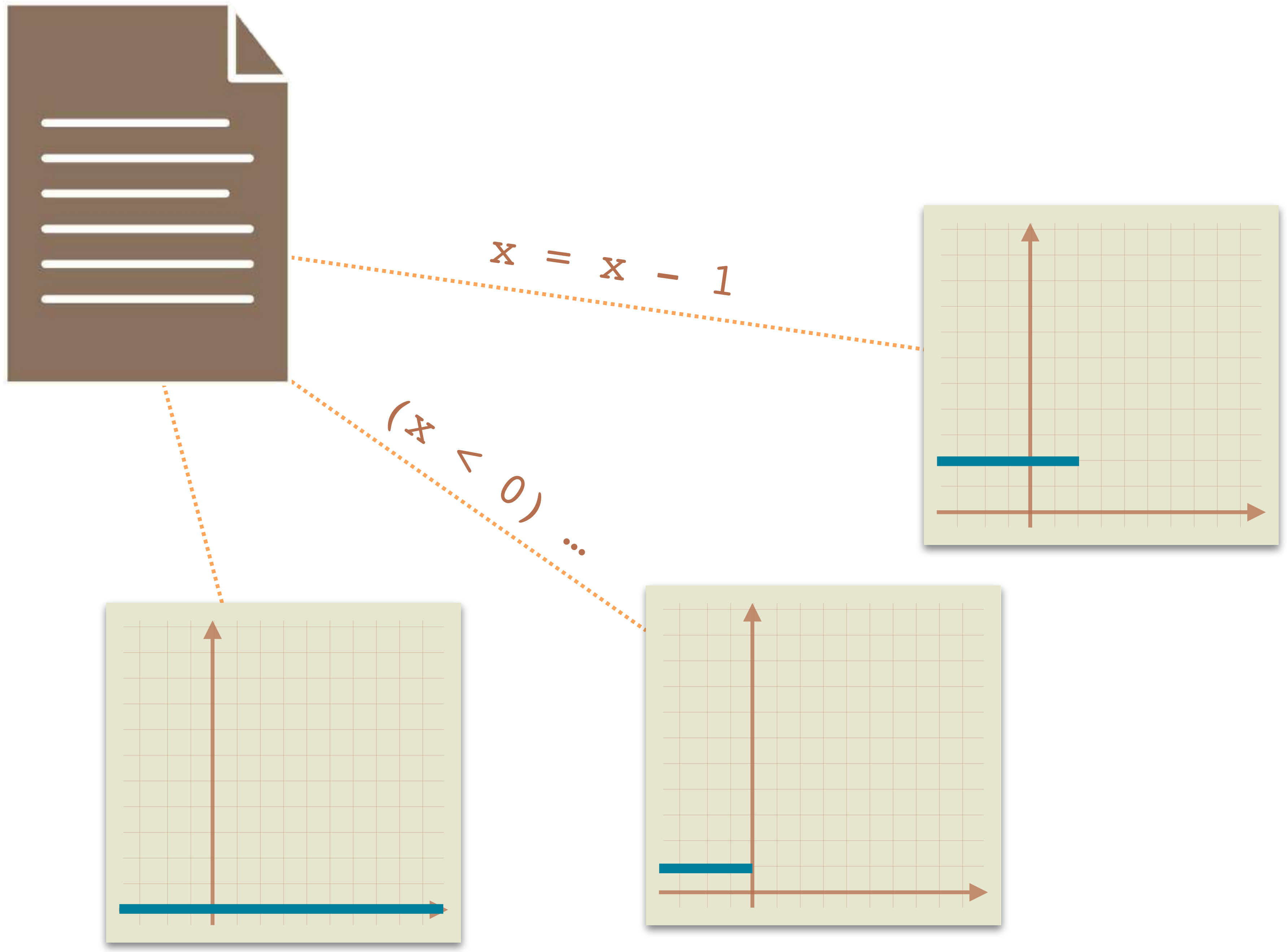
Boolean Conditions

```
function f(x) {

  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
     while 4(z ≥ 0) do
        5z ← z - x
     od6
  else
     while 7(z ≥ x) do
        8c ←[-2, 1]
        9z ←z + c
     od10
  fi

}11
```





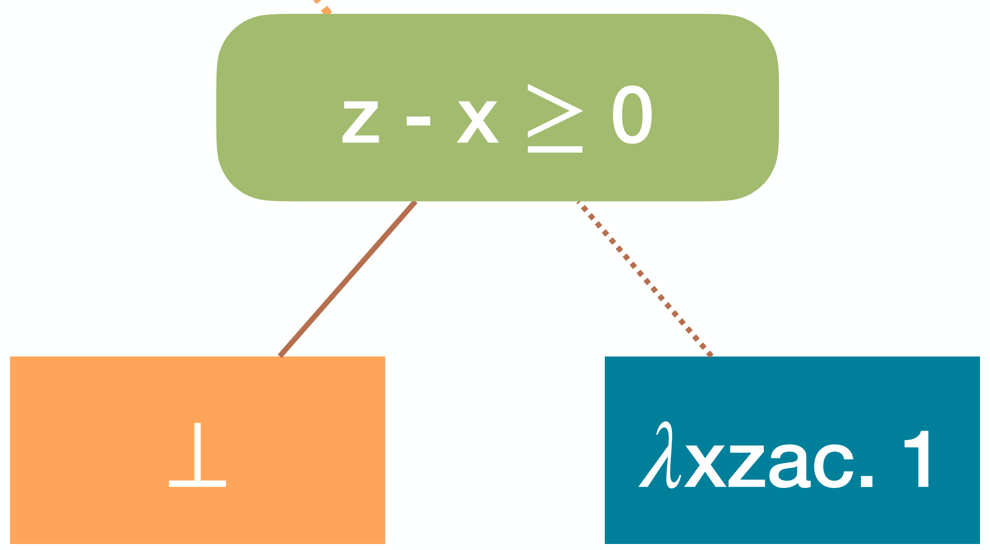
Termination Resilience Static Analysis

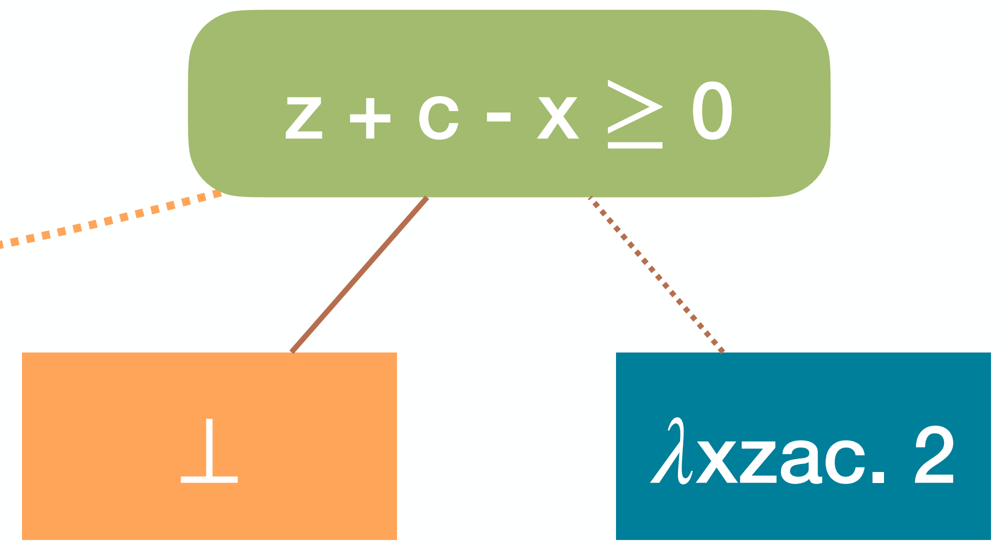


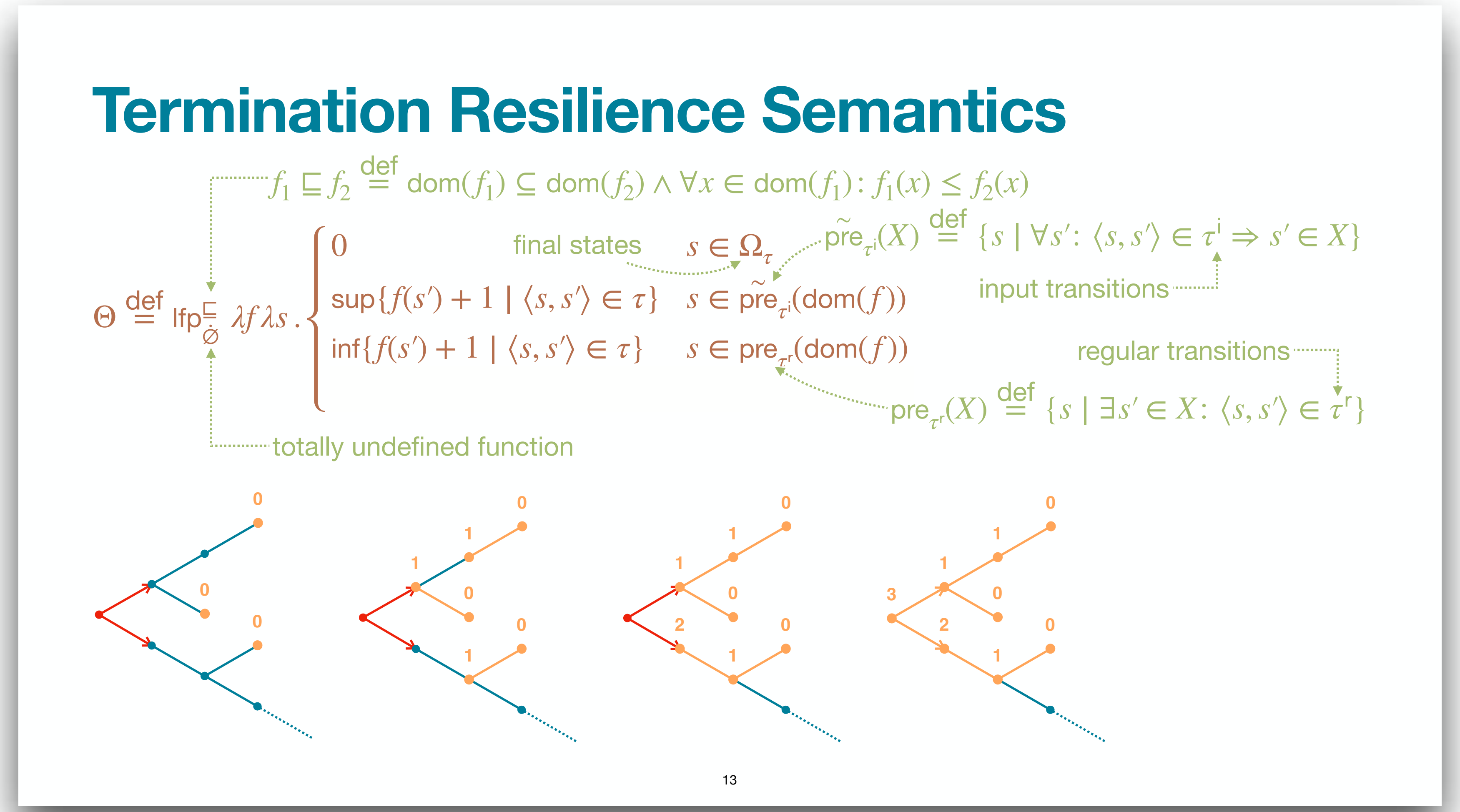
Termination Resilience Static Analysis

Variable Assignment

```
function f(x) {

  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
     while 4(z ≥ 0) do
        5z ← z - x
     od6
  else
     while 7(z ≥ x) do
        8c ←[-2, 1]
        9z ←z + c
     od10
  fi

}11
```



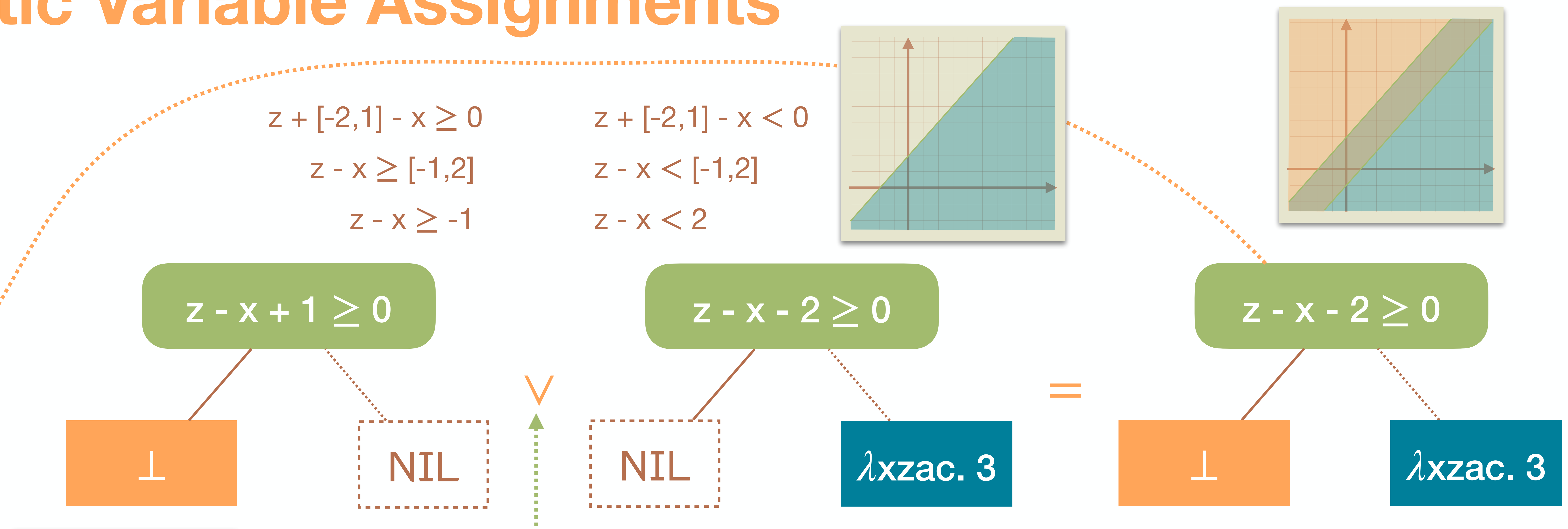


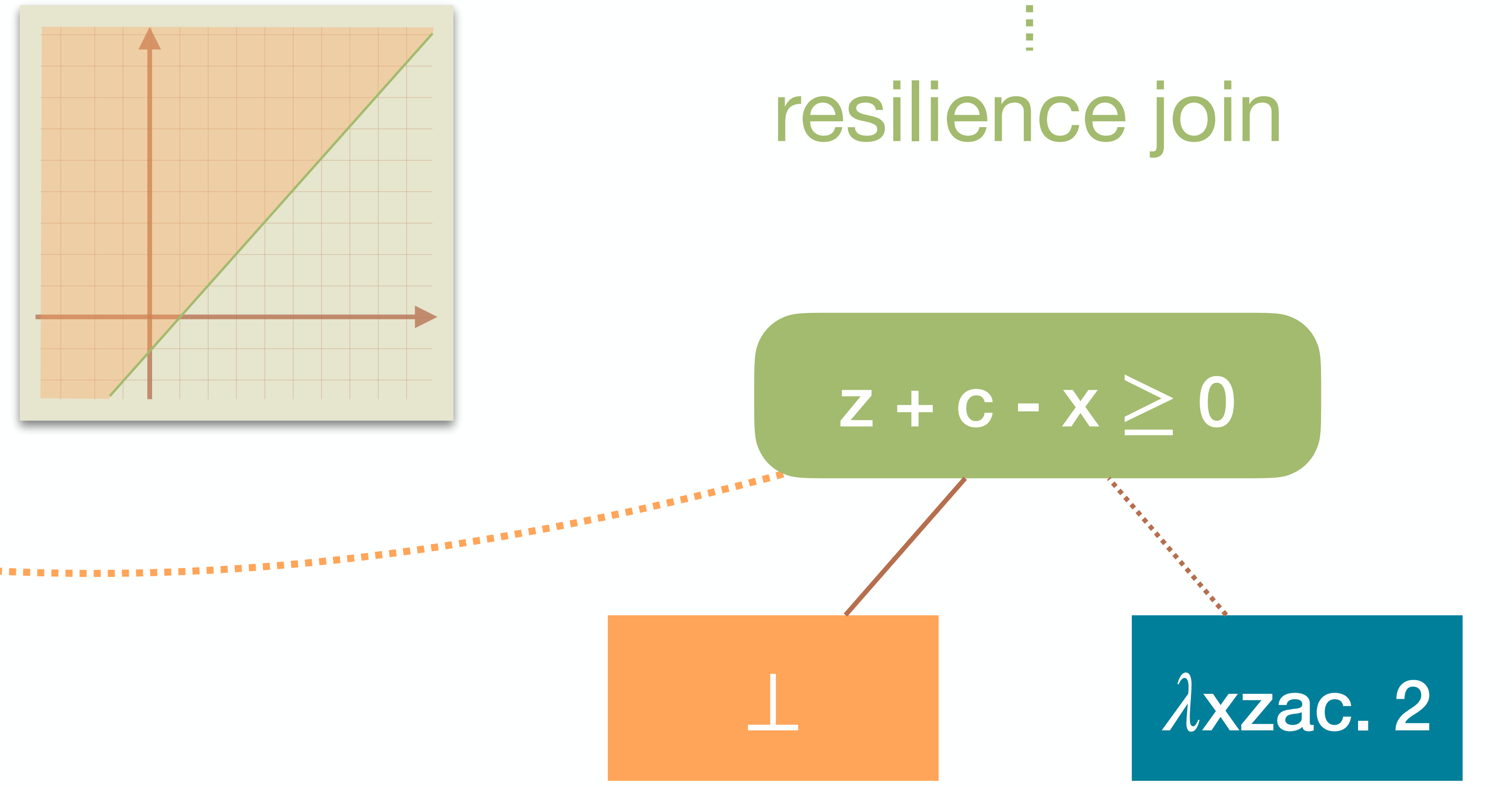


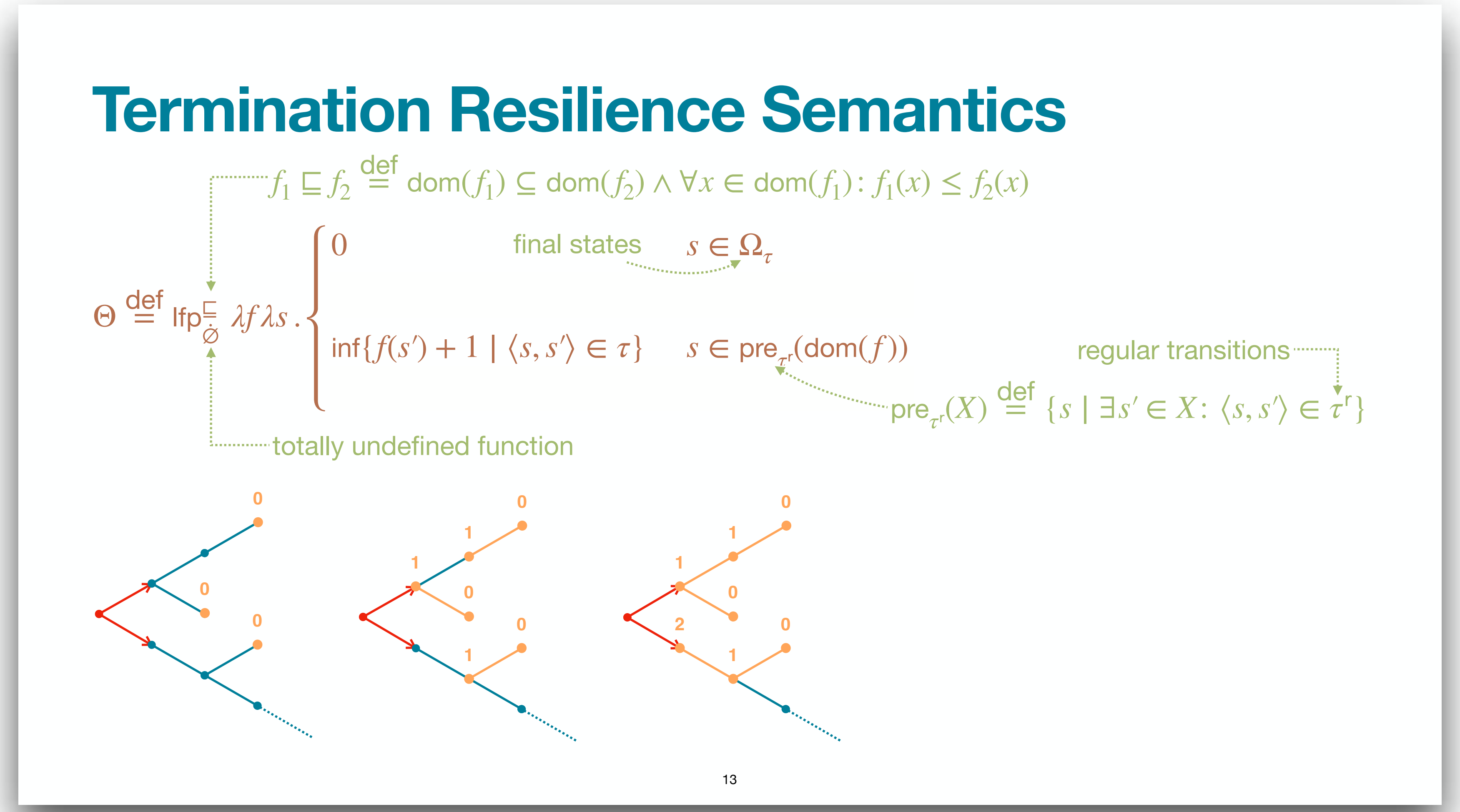
Termination Resilience Static Analysis

Non-Deterministic Variable Assignments

```
function f(x) {

  ¹a ← [-∞, +∞]
  ²z ← 10
  ³if (a*a ≥ 0) then
     while ⁴(z ≥ 0) do
        ⁵z ← z - x
     od⁶
  else
     while ⁷(z ≥ x) do
        ⁸c ←[-2, 1]
        ⁹z ←z + c
     od¹⁰
  fi

}¹¹
```

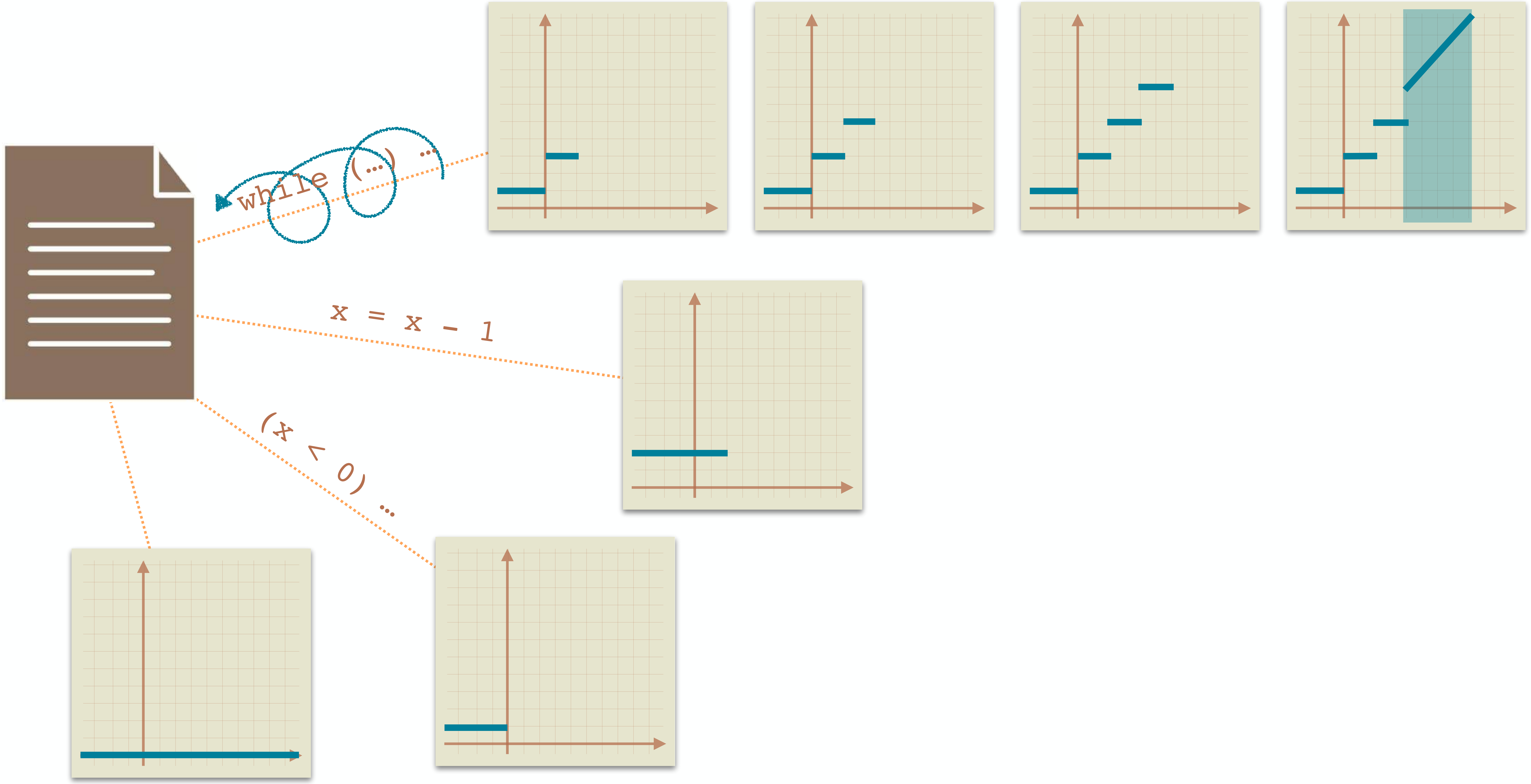






Termination Resilience Static Analysis

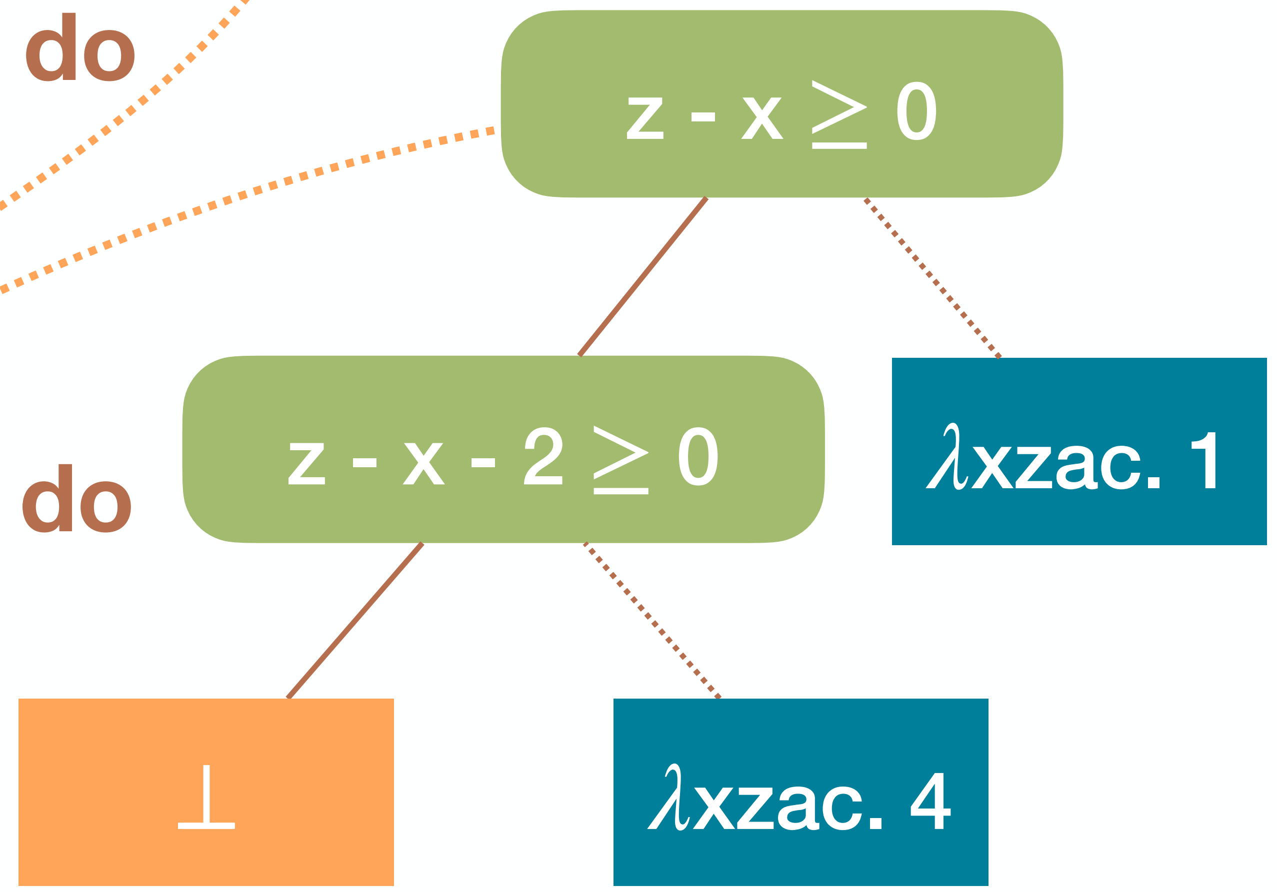
the analysis tries to **predict** a valid ranking function

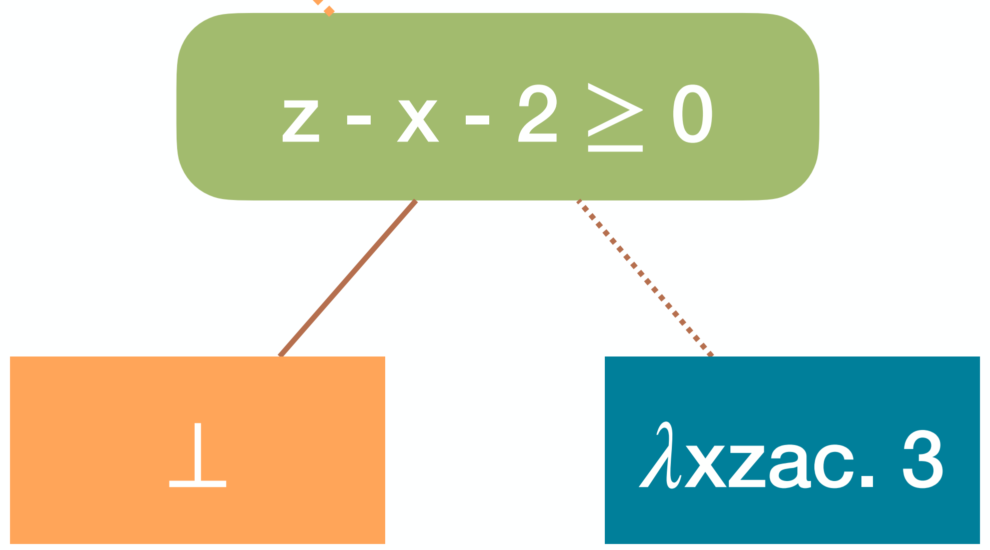


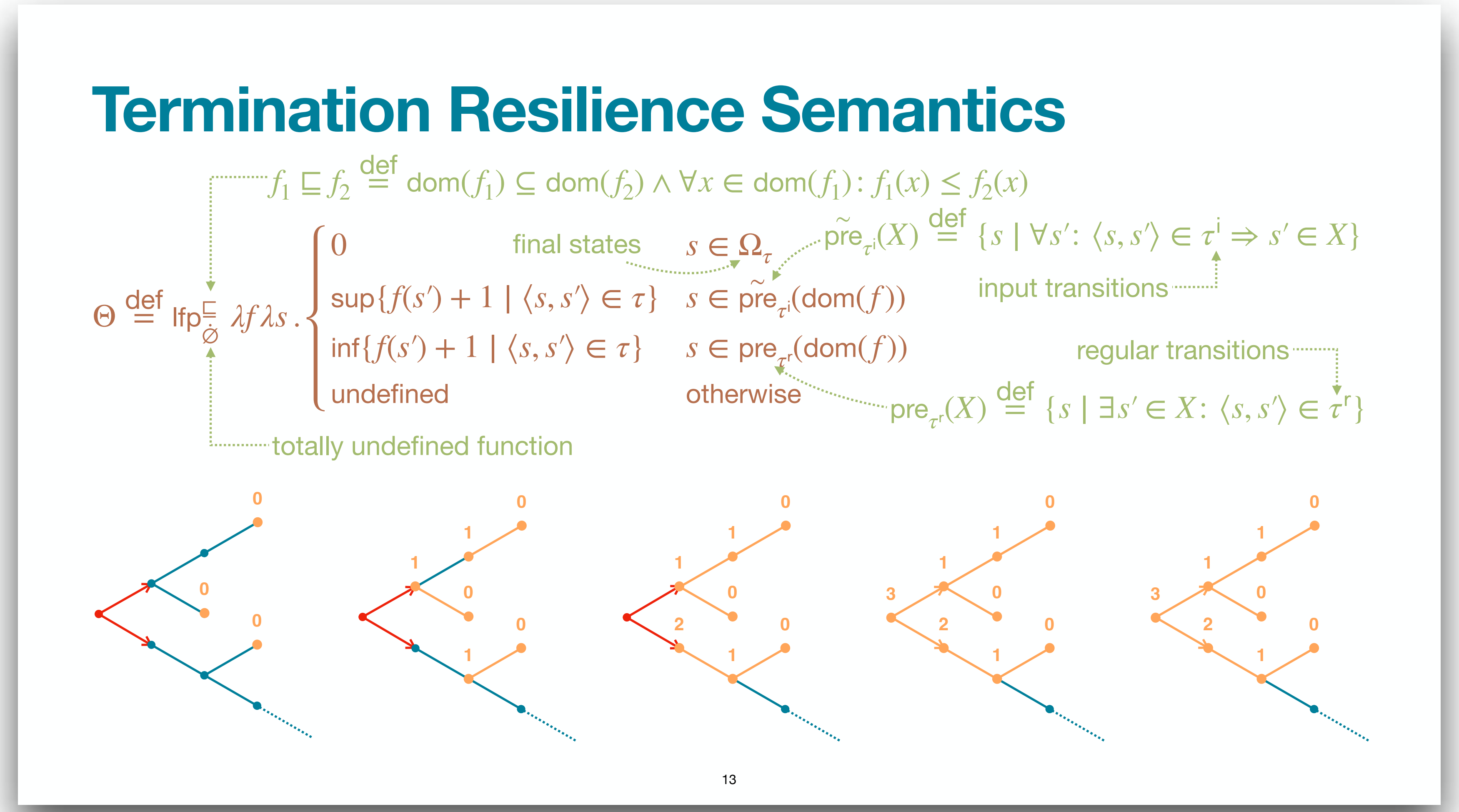
Termination Resilience Static Analysis

Loops

```
function f(x) {

  1 a ← [-∞, +∞]
  2 z ← 10
  3 if (a*a ≥ 0) then
      while 4 (z ≥ 0) do
        5 z ← z - x
      od 6
  else
      while 7 (z ≥ x) do
        8 c ← [-2, 1]
        9 z ← z + c
      od 10
  fi

} 11
```







Termination Resilience Static Analysis

Loops

```
function f(x) {

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  fi

}11
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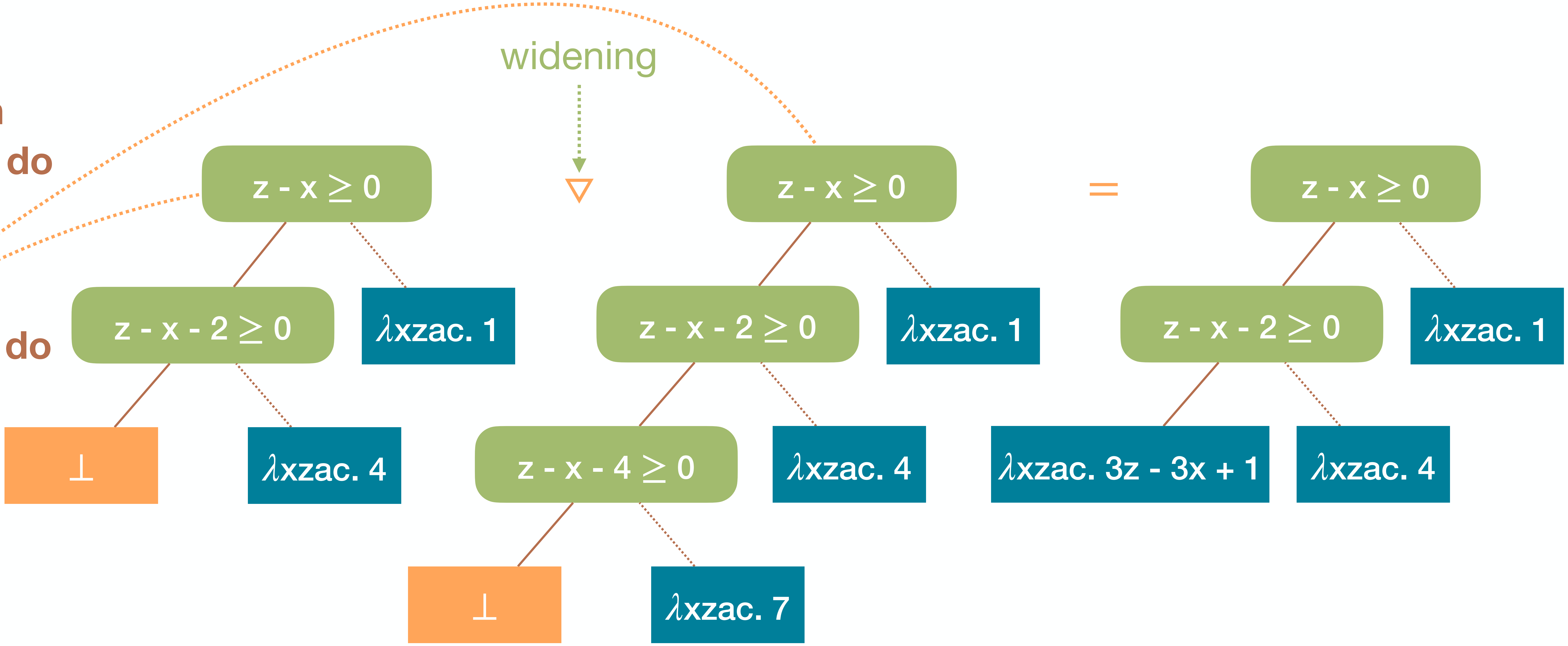

Termination Resilience Static Analysis

Loops

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  fi

}¹¹
```
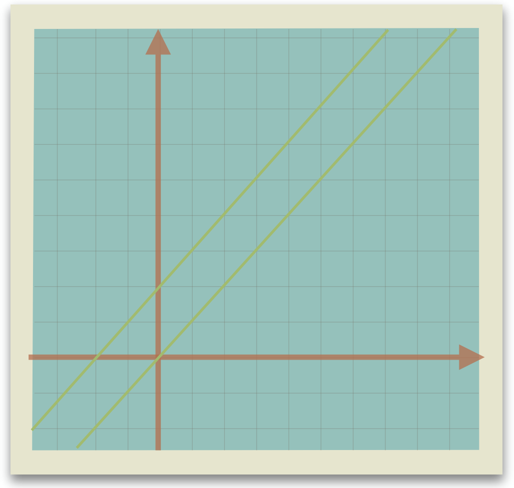
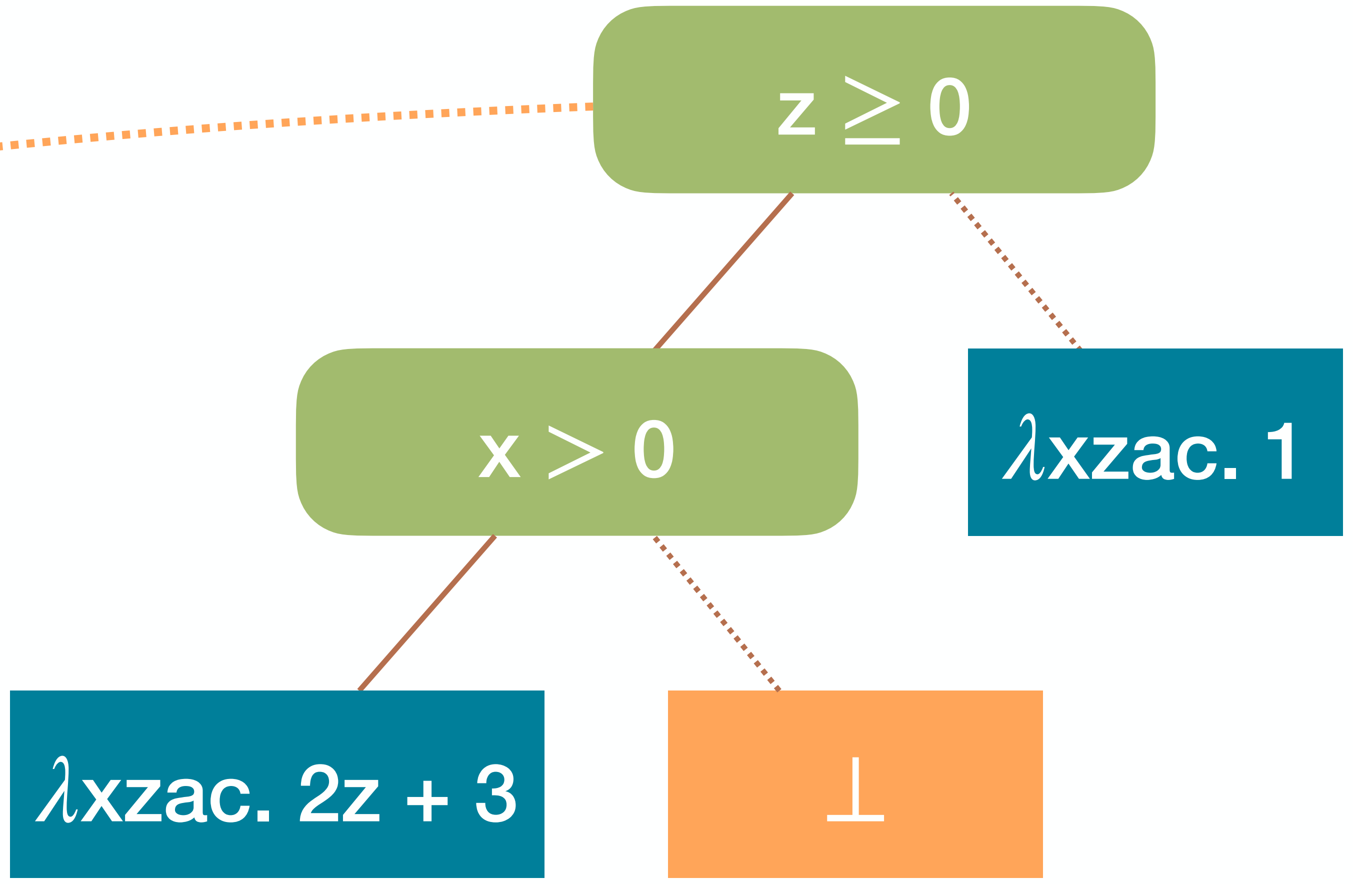
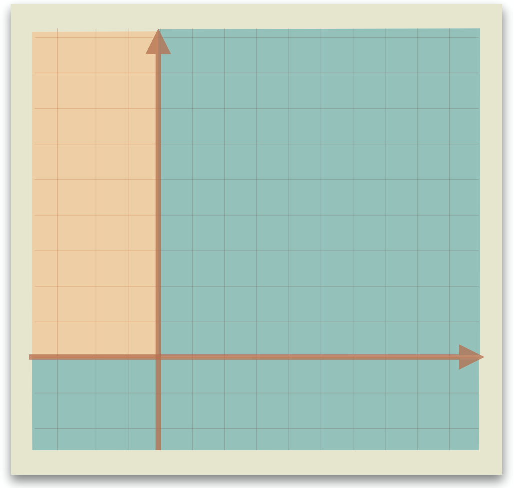
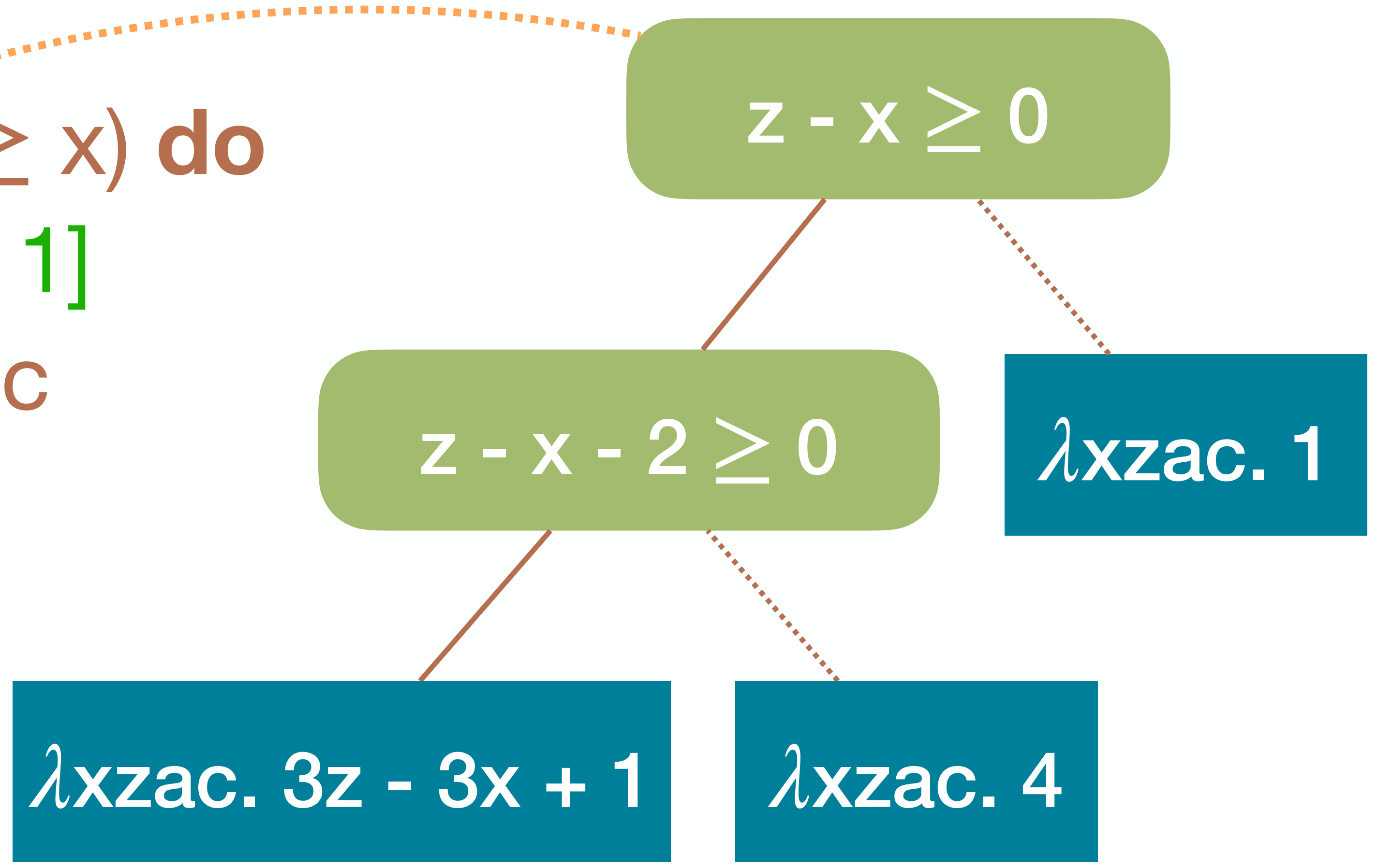
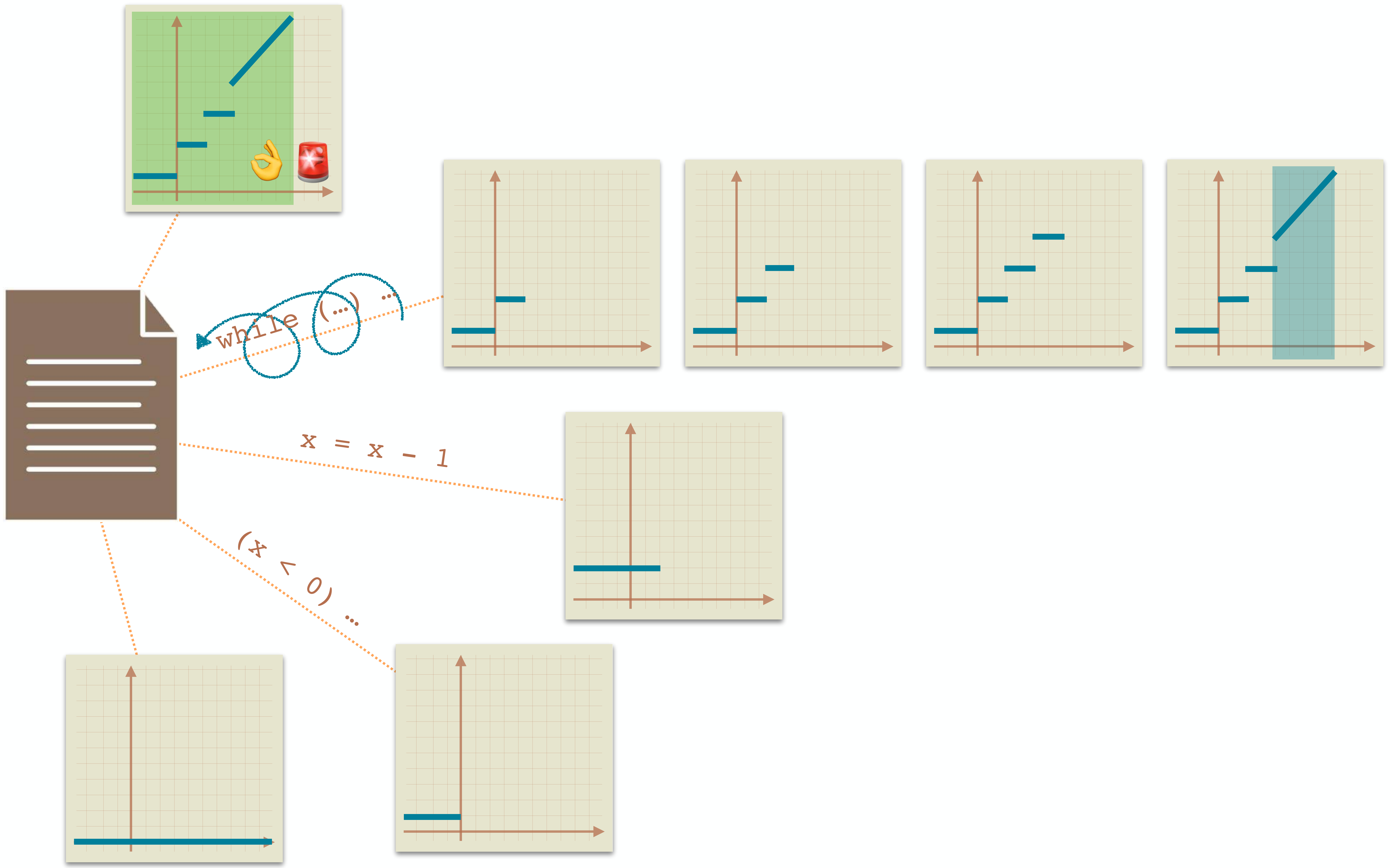









Termination Resilience Static Analysis



Termination Resilience Static Analysis

Approximation Join or Resilience Join?

```
function f(x) {

  ¹a ← [-∞, +∞]
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  ³if (a*a ≥ 0) then
      while ⁴(z ≥ 0) do
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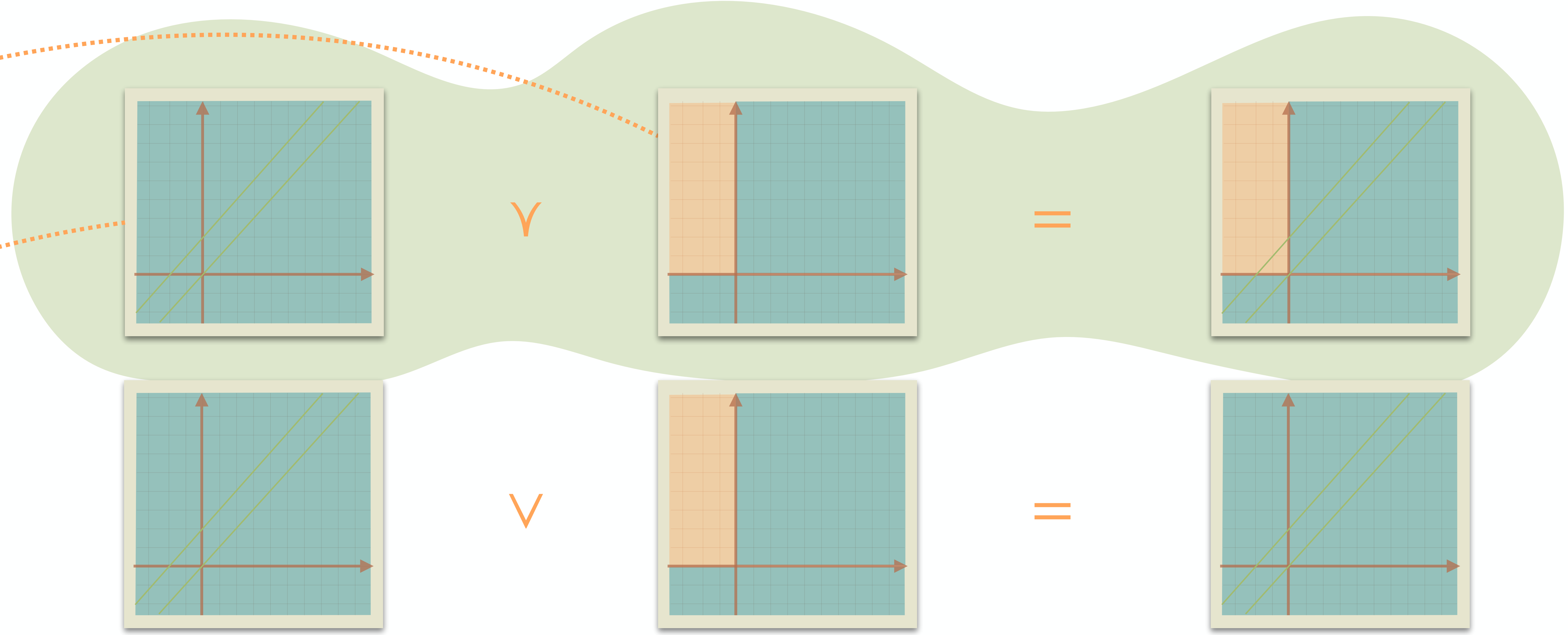

Termination Resilience Static Analysis

```
function f(x) {

  [1]a ← [-∞, +∞]
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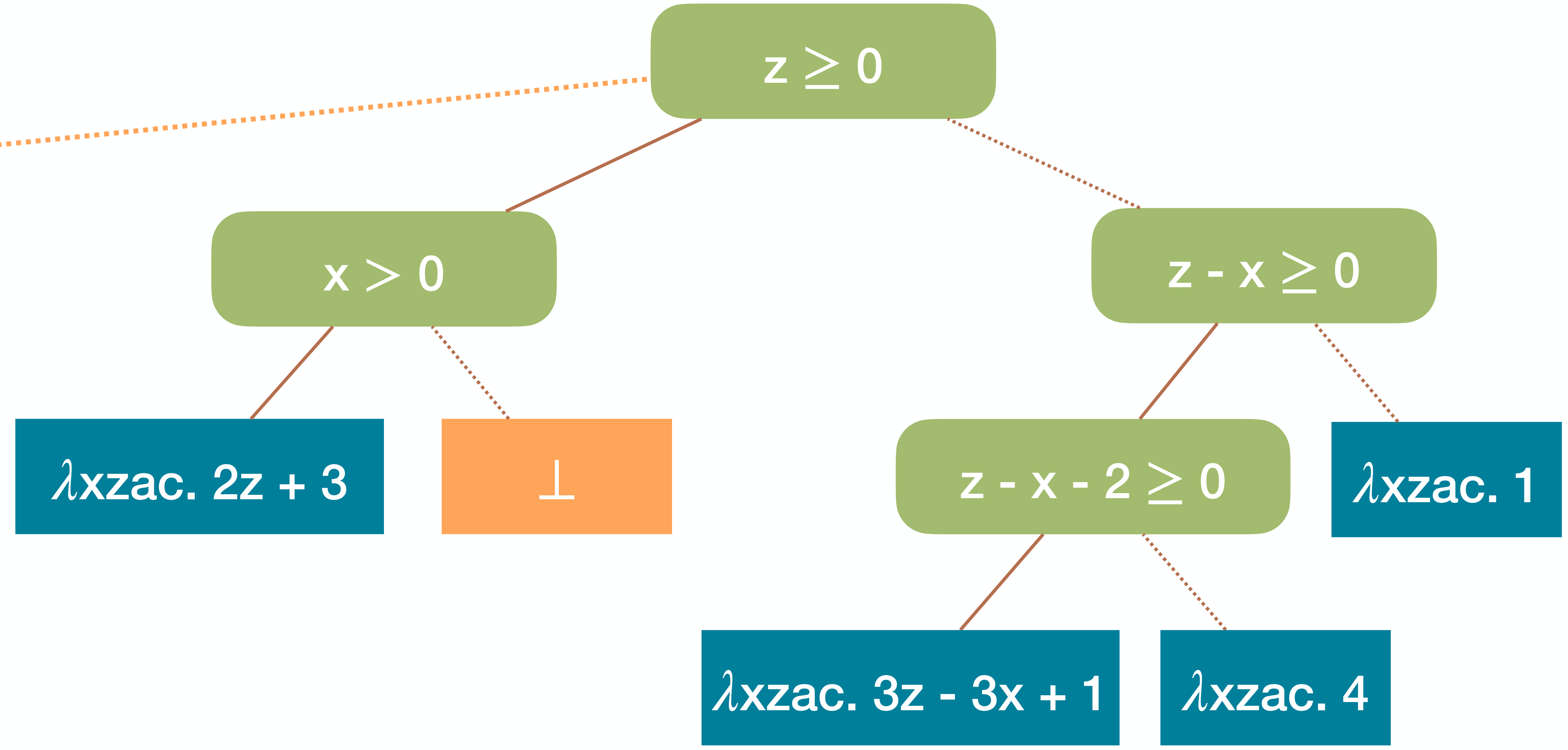

Termination Resilience Static Analysis

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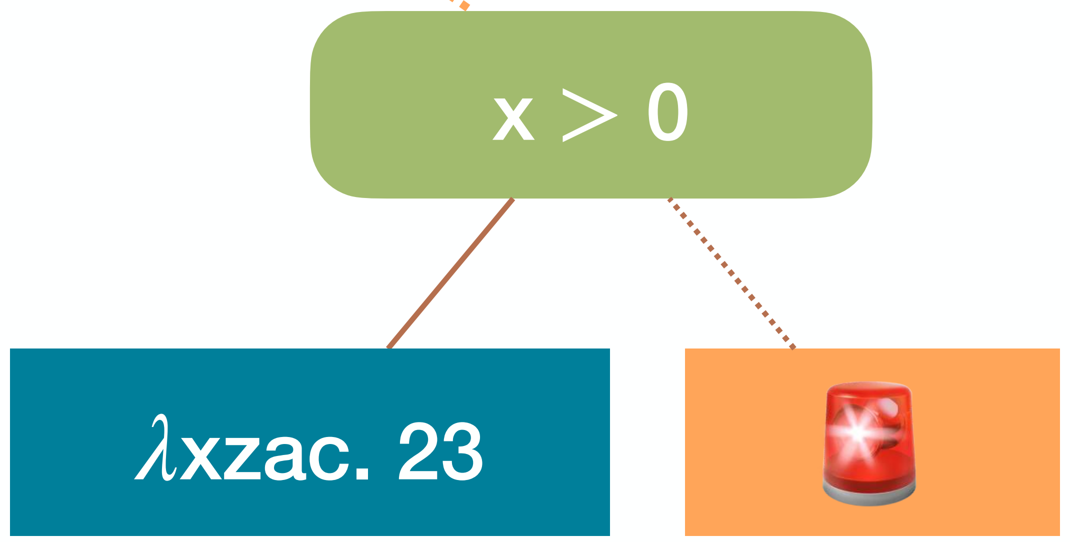

Termination Resilience Static Analysis

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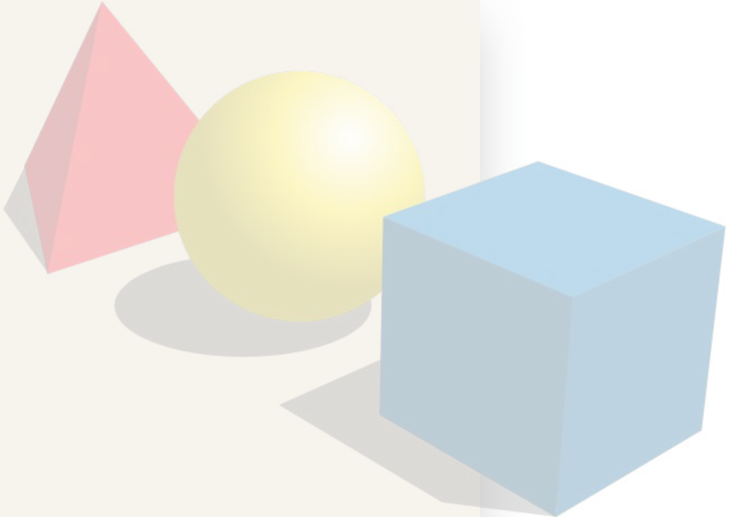


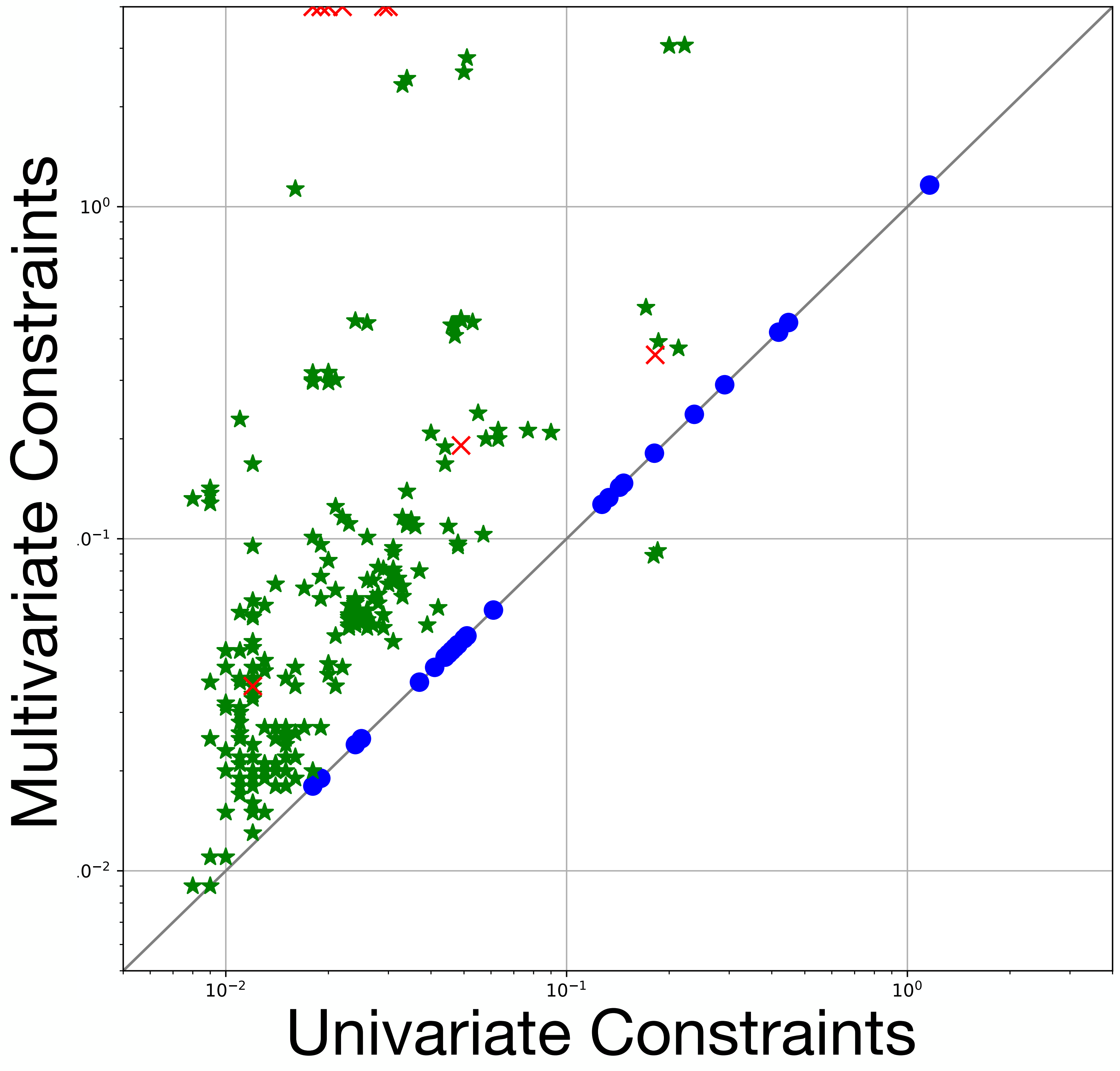
concrete semantics
mathematical models of the program behavior

Experimental Evaluation

	Benchmark	Property	Verified	Alarms	TO	Time
Univariate Constraints	SV-COMP 2024	Termination	0	119	0	3.5s
		Termination Resilience	61	58	0	3.6s
	Raad et al @ OOPSLA 2024	Termination	0	36	0	0.5s
		Termination Resilience	16	20	0	0.5s
	Shi et al. @ FSE 2022	Termination	0	85	0	2.0s
		Termination Resilience	57	28	0	2.2s
Multivariate Constraints	Benchmark	Property	Verified	Alarms	TO	Time
	SV-COMP 2024	Termination	0	119	0	7.2s
		Termination Resilience	76	43	0	16.9s
	Raad et al @ OOPSLA 2024	Termination	0	36	0	7.2s
		Termination Resilience	16	20	0	16.9s
	Shi et al. @ FSE 2022	Termination	0	85	0	69s
		Termination Resilience	49	28	8	500s

Experimental Evaluation

Univariate vs Multivariate Constraints



★ equal precision
× multivariate constraints are more precise
● univariate constraints are more precise (!)

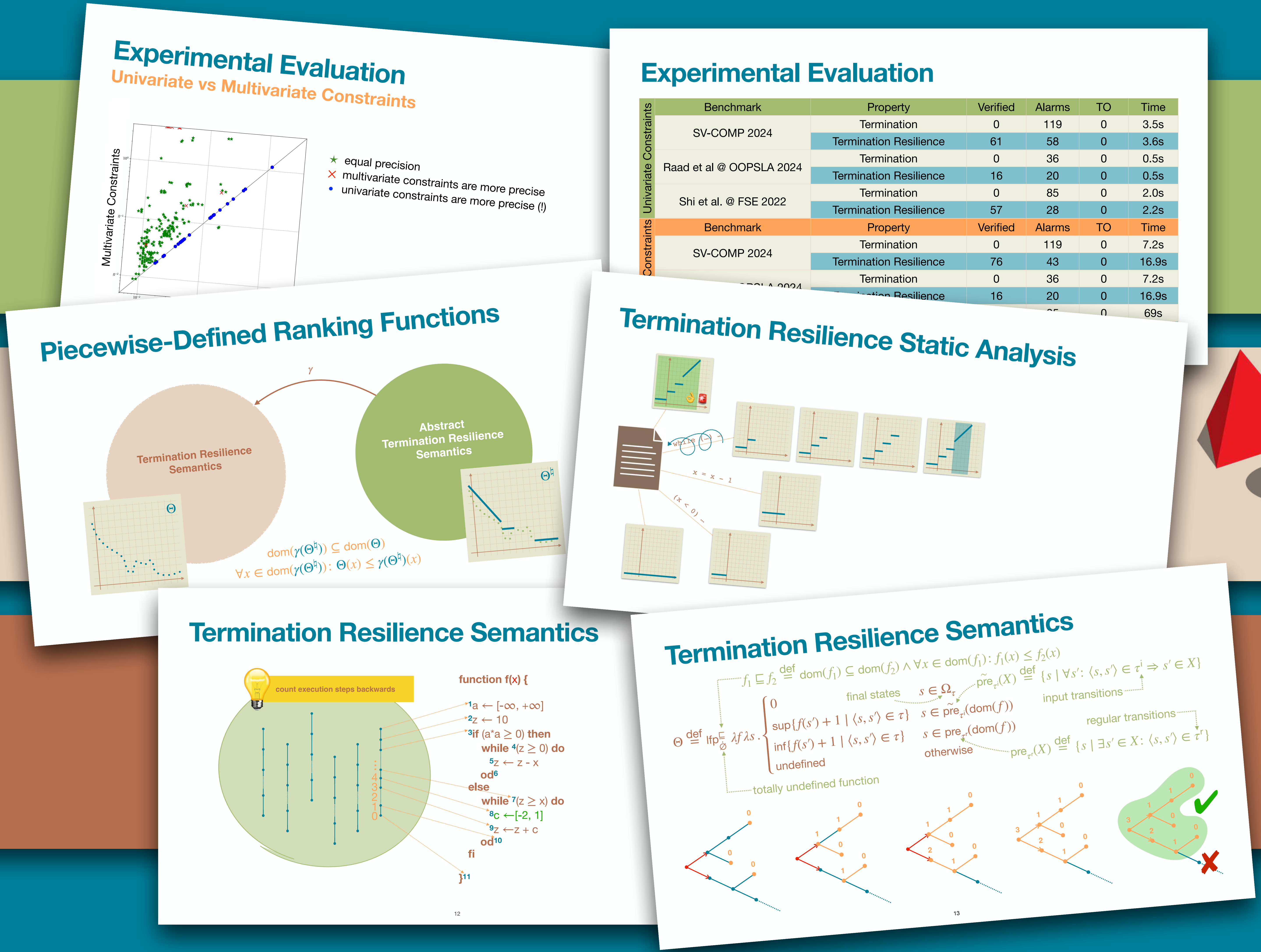
Termination Resilience Static Analysis

3-Step Recipe

practical tools

abstract semantics
abstract domains

concrete semantics



THANKS!