Termination (Resilience) Analysis, and Bugs in Its Implementation

Dagstuhl Seminar 25242 “Testing Program Analyzers and Verifiers”

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Inria & École Normale Supérieure | Université PSL

Which Non-Termination Alarm is Worse?

```
function f(x) {

  1...
  2z ← 10
  3if ( ... ) then
      while 4(z ≥ 0) do
        5z ← z - x
      od6
  else
      while 7(z ≥ x) do
        8c ←[-2, 1]
        9z ←z + c
      od10
  fi

}11
```

diverges when x = 0

diverges when c ≥ 0

non-deterministic value choice

Which Non-Termination Alarm is Worse?

Robust Non-Termination

```
function f(x) {

  1...
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  fi

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```

diverges when x = 0

diverges when c ≥ 0

non-deterministic value choice

Robust Non-Termination

∃ Input ∀ Non-Deterministic Choices : Program Diverges

```
function f(x){                                  demonic non-determinism
  1...
  2z ← 10
  3if ( ... ) then
      while 4(z ≥ 0) do                         🚨 ← diverges when x = 0
        5z ← z - x
      od6
  else
      while 7(z ≥ x) do
        8c ←[-2, 1]
        9z ←z + c
      od10
  fi

}11
```

Termination Resilience

∀ Inputs ∃ Non-Deterministic Choice : Program Terminates

```
function f(x) {

  1...
  2z ← 10
  3if ( ... ) then
      while 4(z ≥ 0) do
        5z ← z - x
      od6
  else
      while 7(z ≥ x) do
        8c ←[-2, 1]
        9z ←z + c
      od10
  fi

}11
```

👌 ⟵ terminates when **c < 0, independently of the value of x**

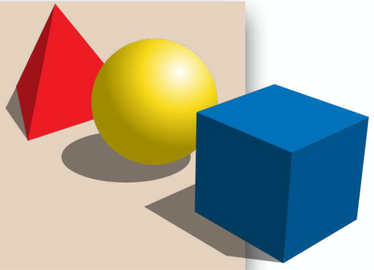
angelic non-determinism

Termination Resilience Static Analysis

3-Step Recipe

practical tools
targeting specific programs

abstract semantics, abstract domains
algorithmic approaches to decide program properties



concrete semantics
mathematical models of the program behavior

Termination Resilience Static Analysis

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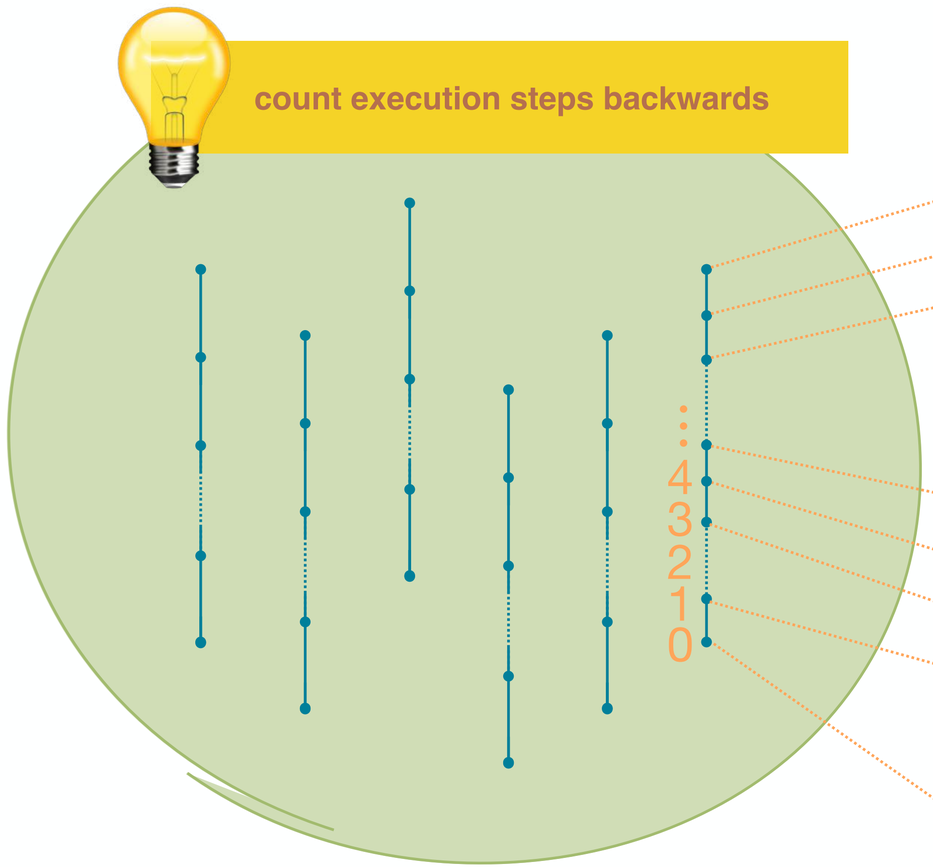
Termination Resilience Semantics




```
function f(x) {

  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
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  else
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```

Termination Resilience Semantics

$$f_1 \sqsubseteq f_2 \overset{\text{def}}{=} \mathrm{dom}(f_1) \subseteq \mathrm{dom}(f_2) \wedge \forall x \in \mathrm{dom}(f_1) : f_1(x) \leq f_2(x)$$

$$\Theta \overset{\text{def}}{=} \mathrm{lfp}^{\sqsubseteq}_{\dot{\varnothing}}\ \lambda f\, \lambda s\,.\ \begin{cases} 0 & \text{final states} \quad s \in \Omega_\tau \\ \sup\{f(s') + 1 \mid \langle s, s' \rangle \in \tau\} & s \in \widetilde{\mathrm{pre}}_{\tau^{\mathsf{i}}}(\mathrm{dom}(f)) \\ \inf\{f(s') + 1 \mid \langle s, s' \rangle \in \tau\} & s \in \mathrm{pre}_{\tau^{\mathsf{r}}}(\mathrm{dom}(f)) \\ \text{undefined} & \text{otherwise} \end{cases}$$

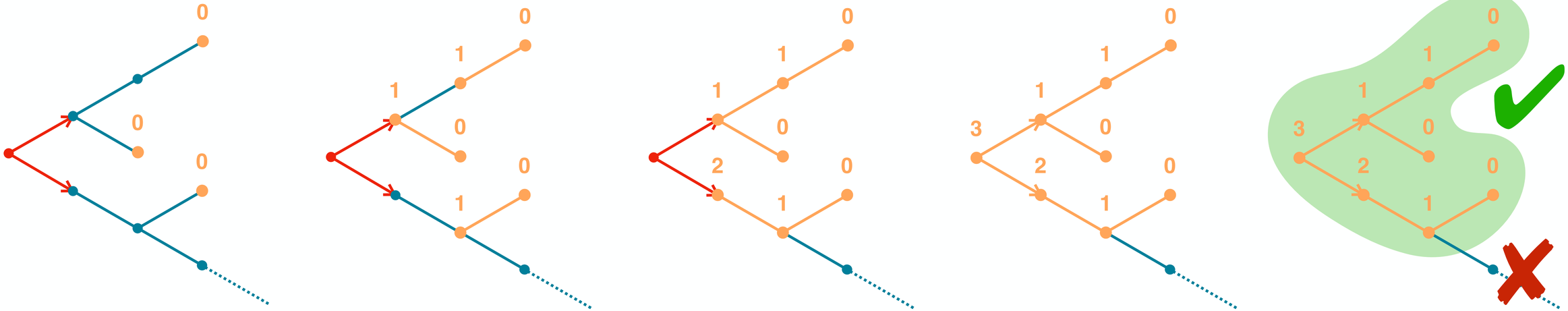
$$\widetilde{\mathrm{pre}}_{\tau^{\mathsf{i}}}(X) \overset{\text{def}}{=} \{s \mid \forall s' \colon \langle s, s' \rangle \in \tau^{\mathsf{i}} \Rightarrow s' \in X\}$$

input transitions

regular transitions

$$\mathrm{pre}_{\tau^{\mathsf{r}}}(X) \overset{\text{def}}{=} \{s \mid \exists s' \in X \colon \langle s, s' \rangle \in \tau^{\mathsf{r}}\}$$

totally undefined function



Termination Resilience Static Analysis

3-Step Recipe

practical tools
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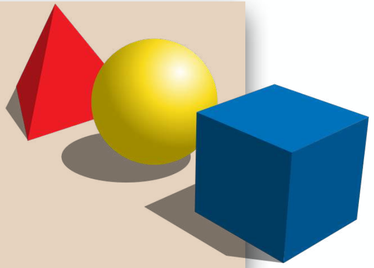
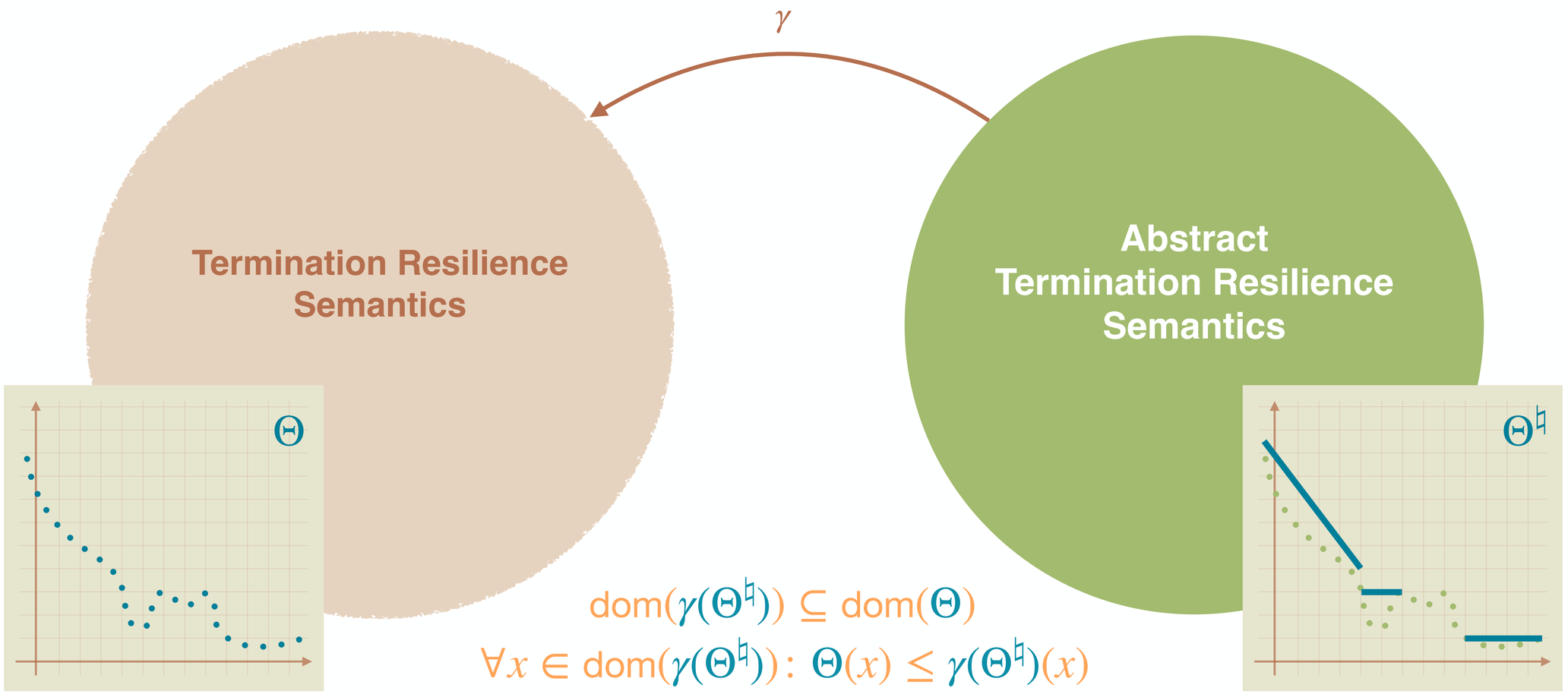
abstract semantics, abstract domains
algorithmic approaches to decide program properties

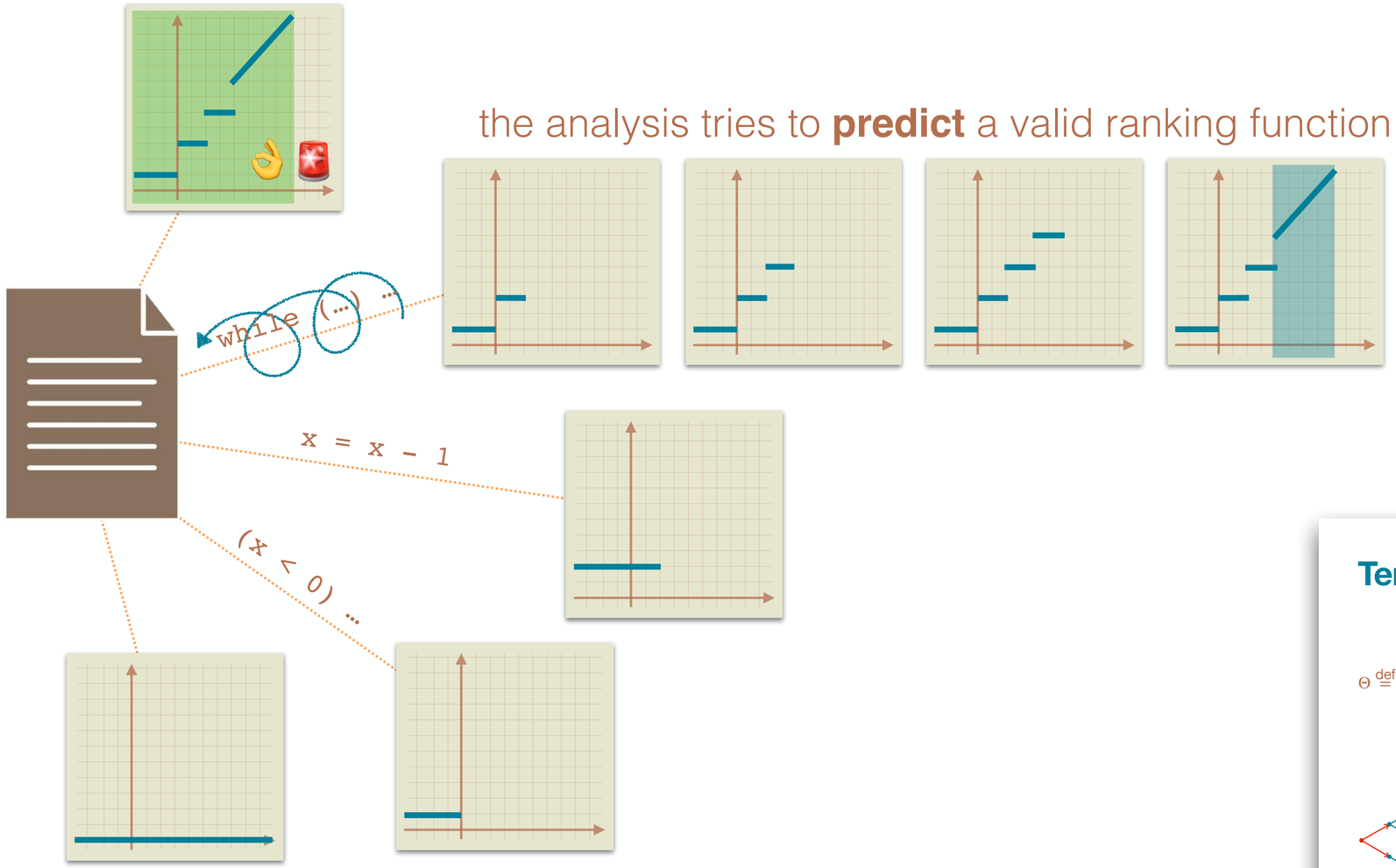


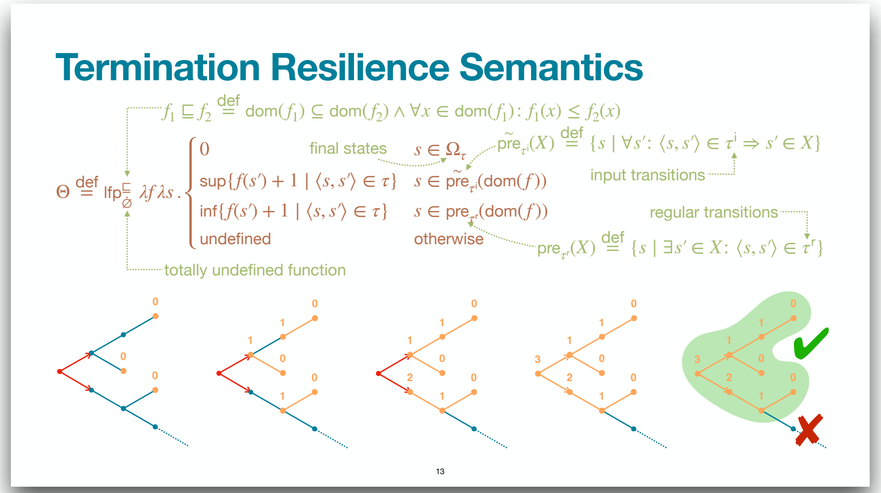
concrete semantics
mathematical models of the program behavior

Piecewise-Defined Ranking Functions

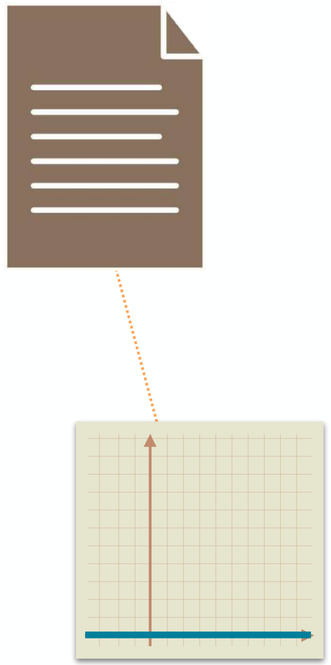


Termination Resilience Static Analysis





Termination Resilience Static Analysis



Termination Resilience Static Analysis

Static Backward Analysis

```
function f(x) {

  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
      while 4(z ≥ 0) do
        5z ← z - x
      od6
  else
      while 7(z ≥ x) do
        8c ←[-2, 1]
        9z ←z + c
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  fi

}11
```
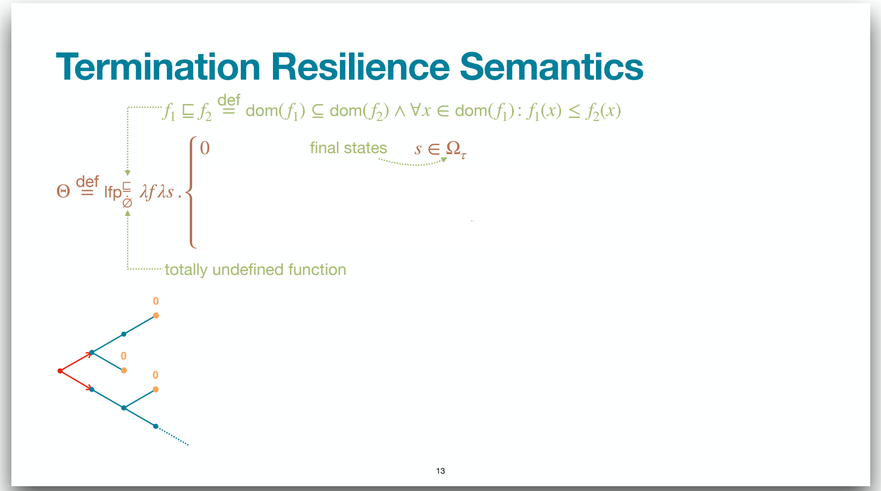
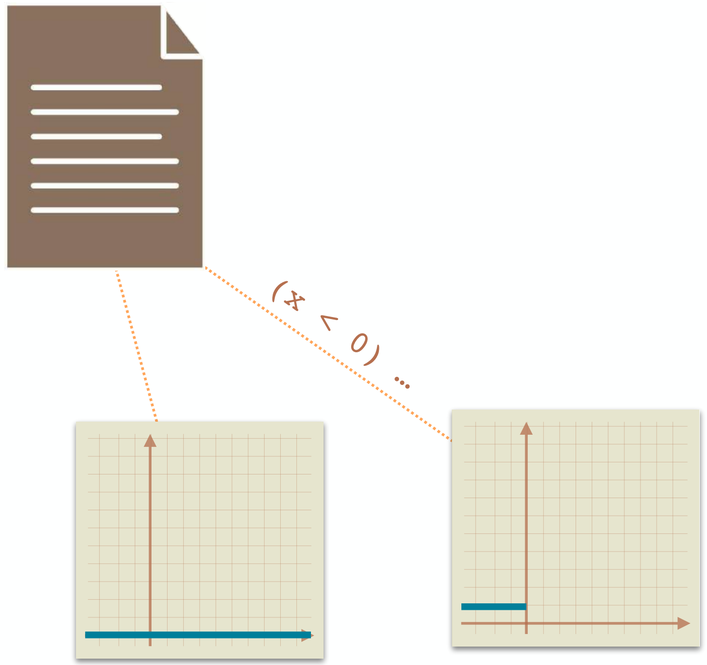

λxzac. 0



Termination Resilience Static Analysis



Termination Resilience Static Analysis

Boolean Conditions

```
function f(x) {
  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
      while 4(z ≥ 0) do
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      od6
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}11
```
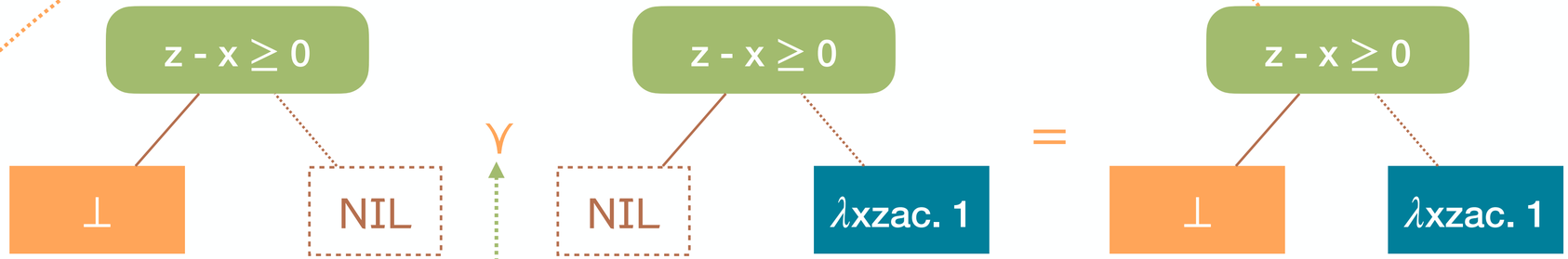
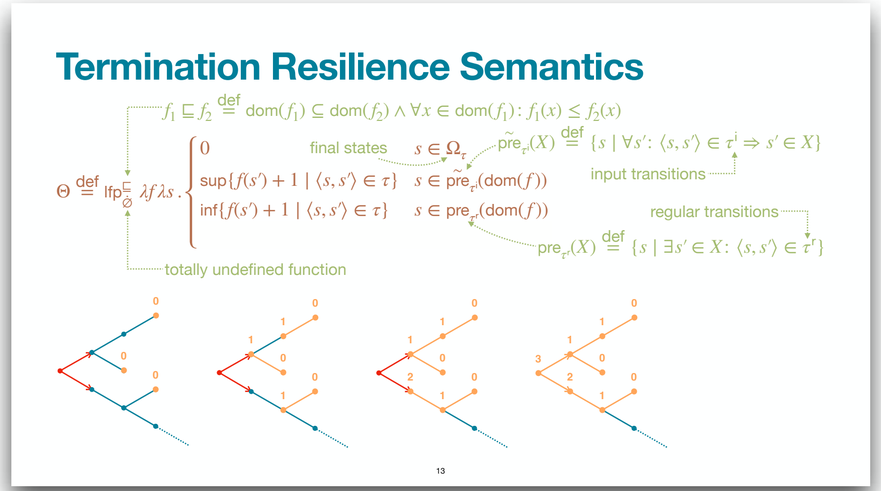
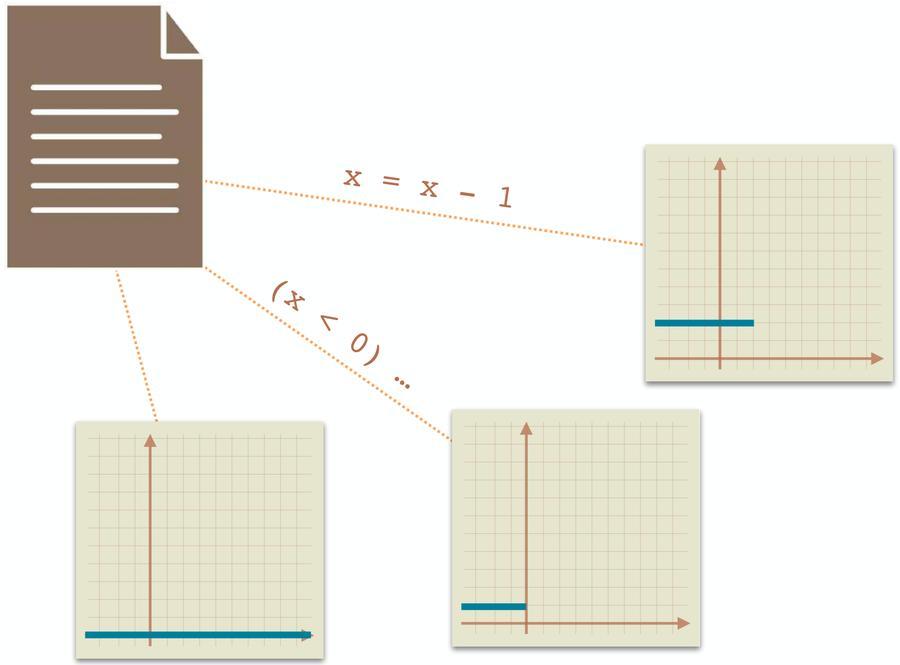



λxzac. 0



Termination Resilience Static Analysis



Termination Resilience Static Analysis

Variable Assignment

```
function f(x) {

  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
      while 4(z ≥ 0) do
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  else
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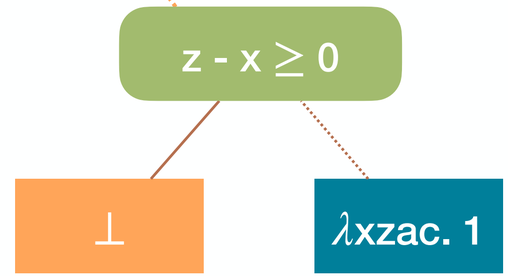

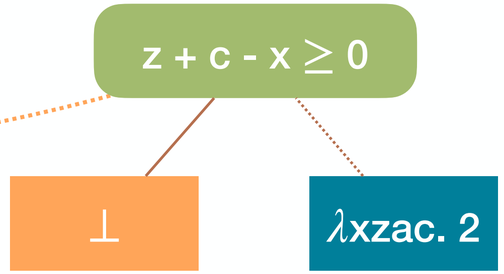

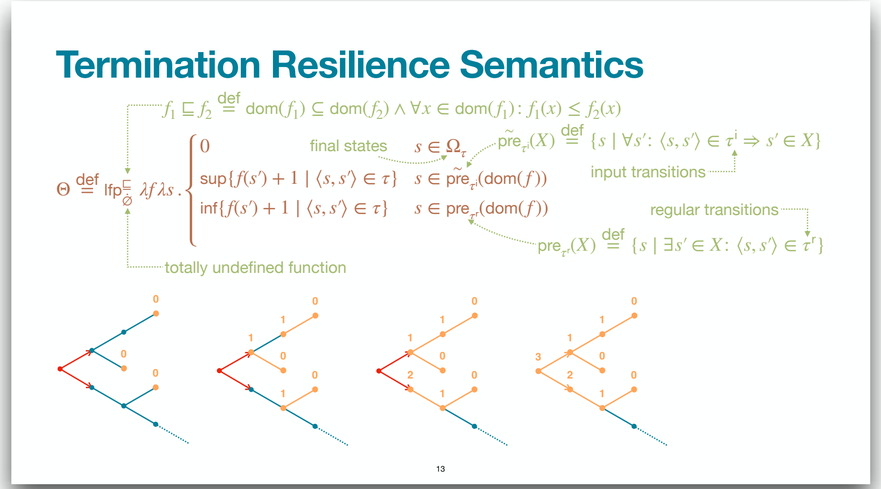

Termination Resilience Static Analysis

Non-Deterministic Variable Assignments

```
function f(x) {

  1a ← [-∞, +∞]
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  3if (a*a ≥ 0) then
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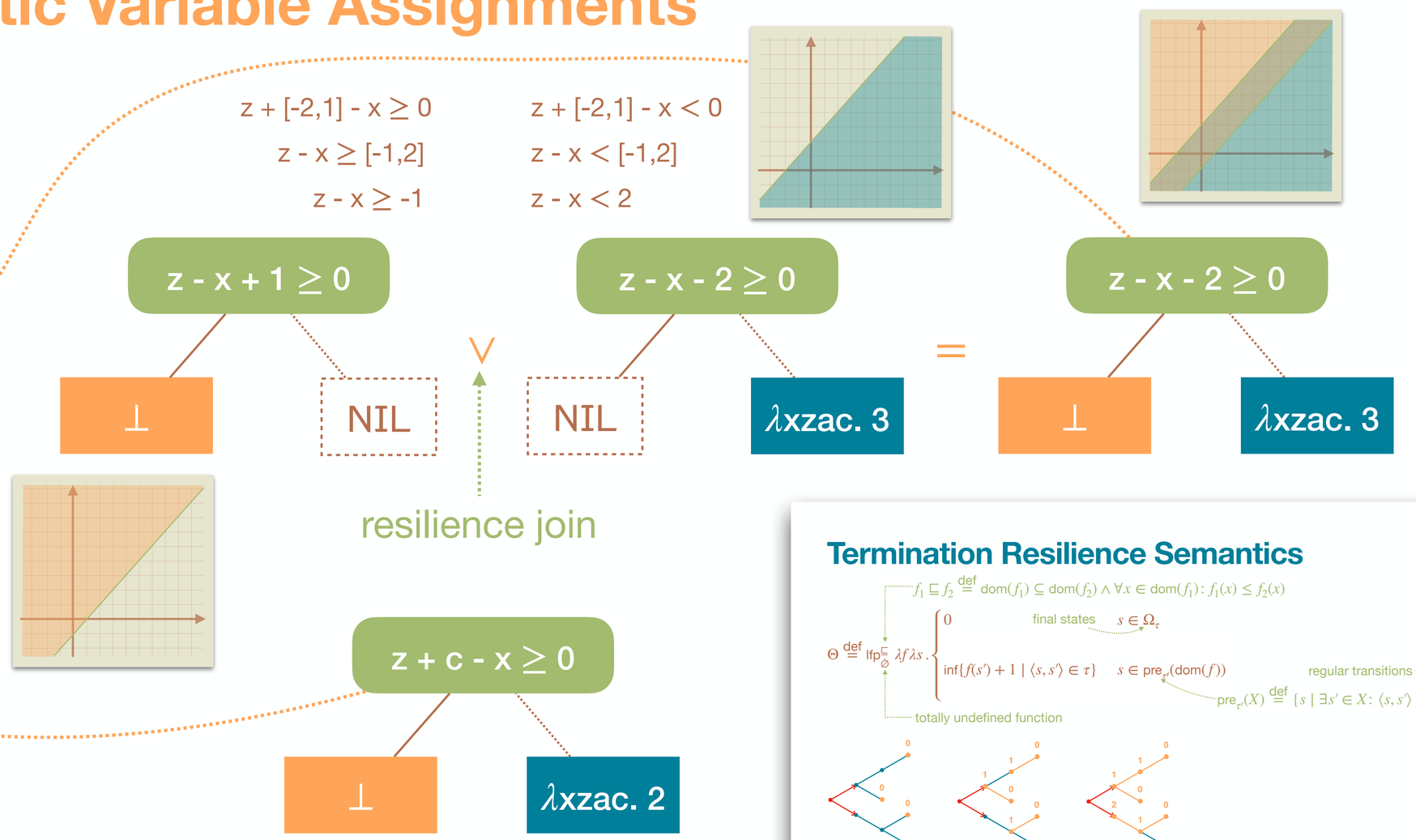


Termination Resilience Semantics

$$f_1 \sqsubseteq f_2 \stackrel{\text{def}}{=} \text{dom}(f_1) \subseteq \text{dom}(f_2) \wedge \forall x \in \text{dom}(f_1) : f_1(x) \leq f_2(x)$$

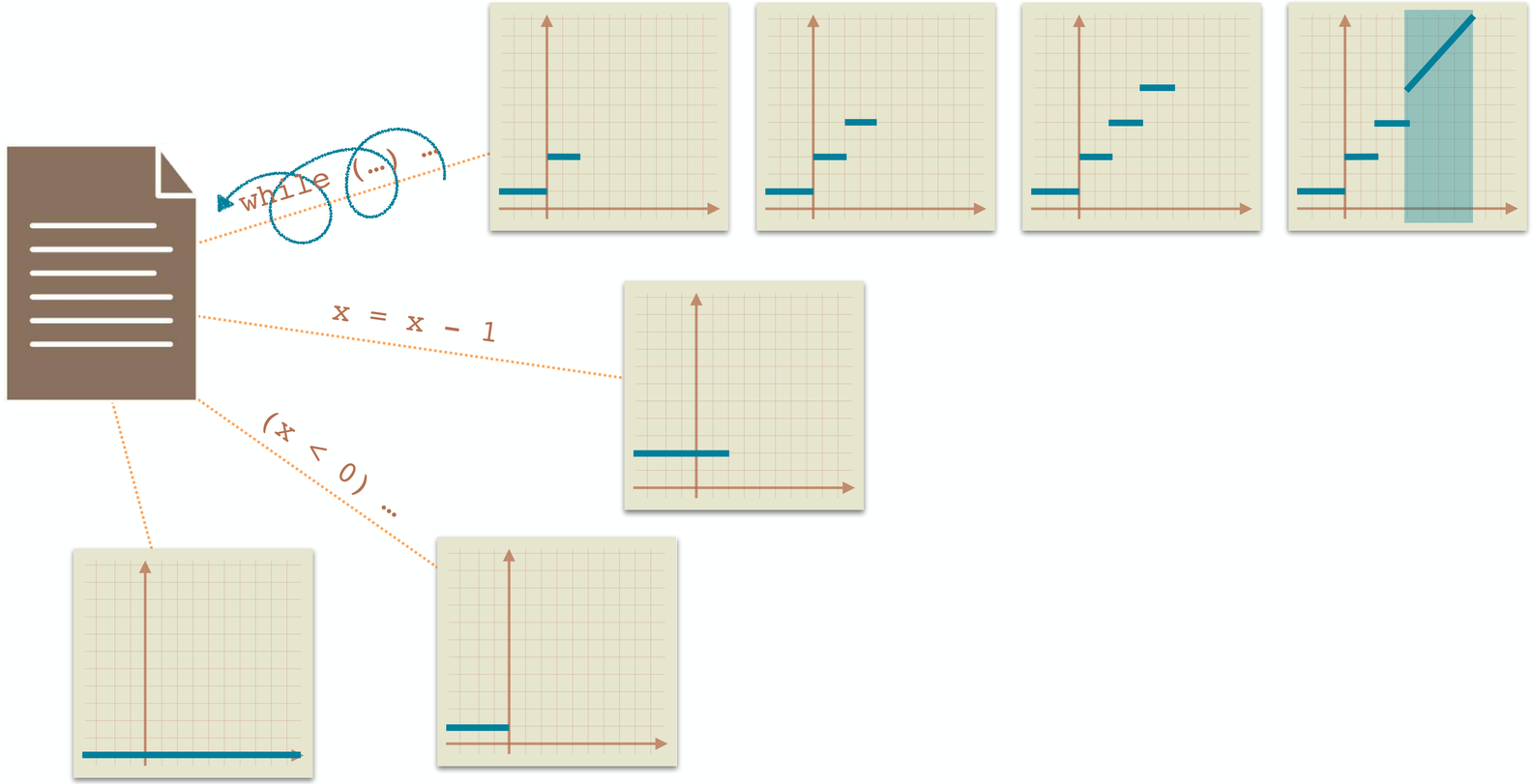
$$\Theta \stackrel{\text{def}}{=} \text{lfp}^{\sqsubseteq}_{\varnothing}\ \lambda f \lambda s .\ \begin{cases} 0 & s \in \Omega_\tau \quad \text{(final states)} \\ \inf\{f(s') + 1 \mid \langle s, s' \rangle \in \tau\} & s \in \text{pre}_\tau(\text{dom}(f)) \quad \text{(regular transitions)} \end{cases}$$

$$\text{pre}_\tau(X) \stackrel{\text{def}}{=} \{s \mid \exists s' \in X : \langle s, s' \rangle \in \tau\}$$

totally undefined function

Termination Resilience Static Analysis

the analysis tries to **predict** a valid ranking function



Termination Resilience Static Analysis

Loops

```
function f(x) {

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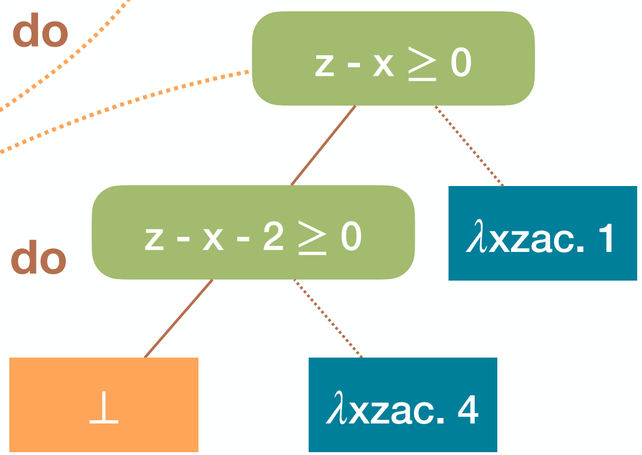

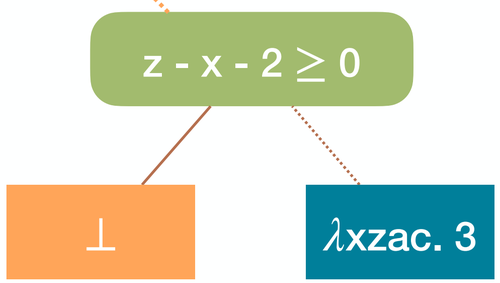

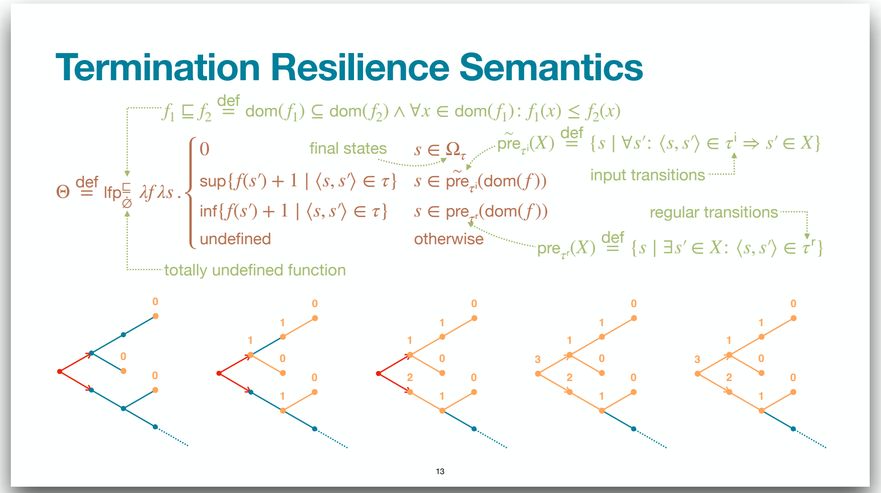

Termination Resilience Static Analysis

Loops

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function f(x) {
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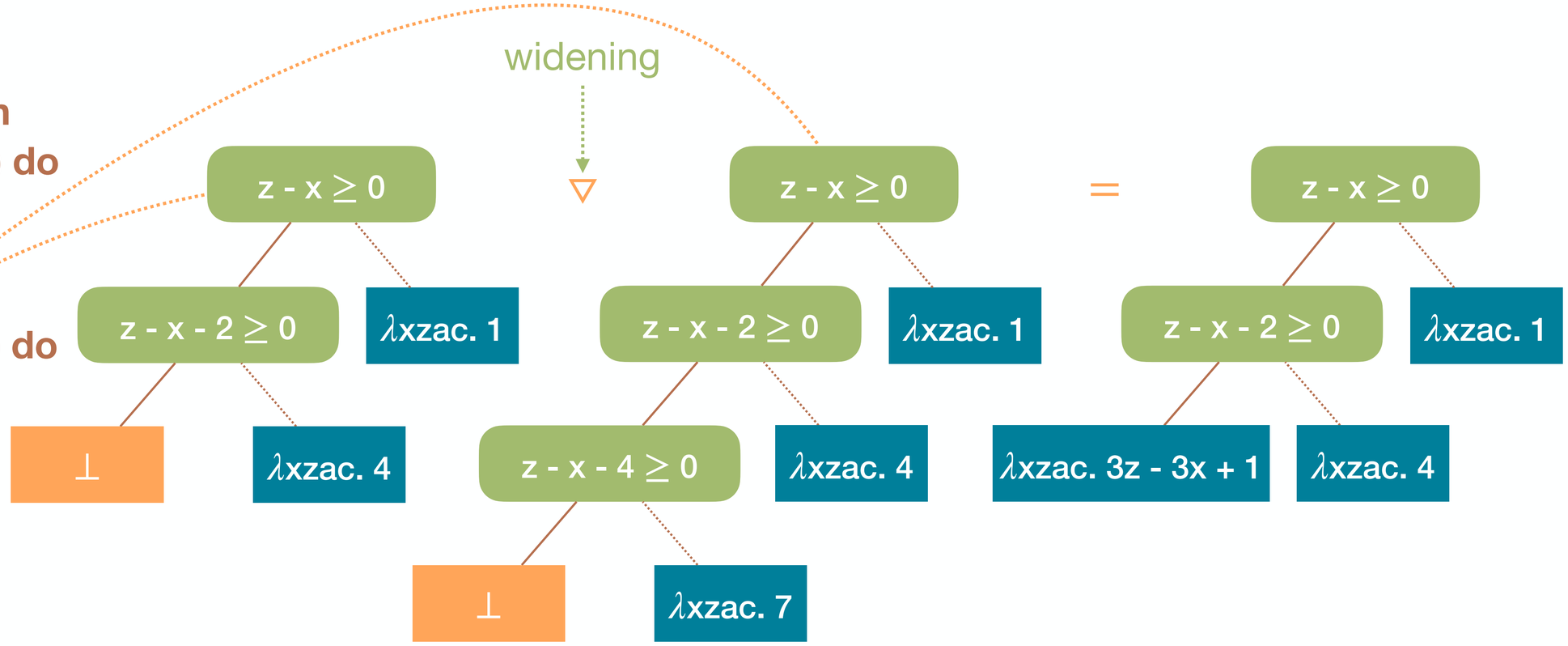

Termination Resilience Static Analysis

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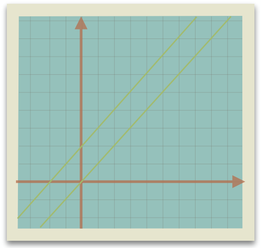
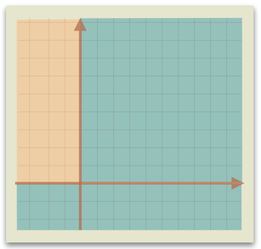
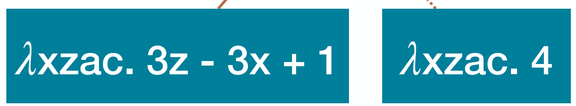
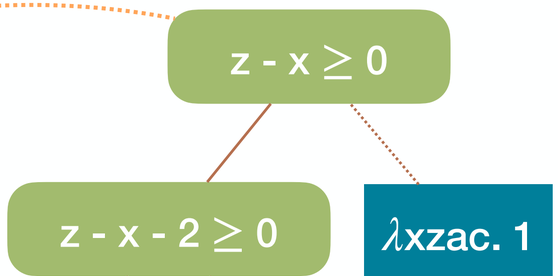
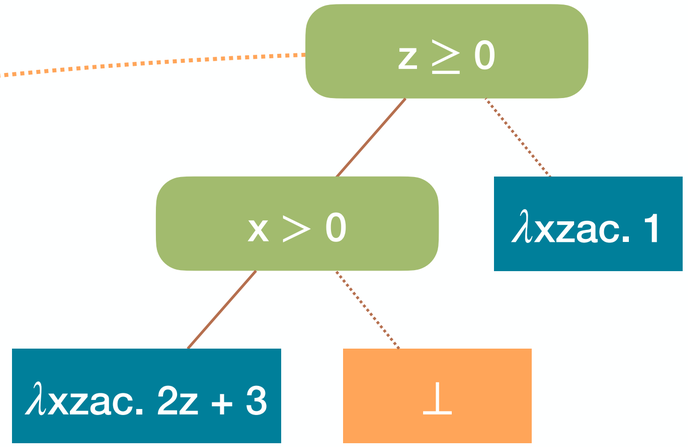


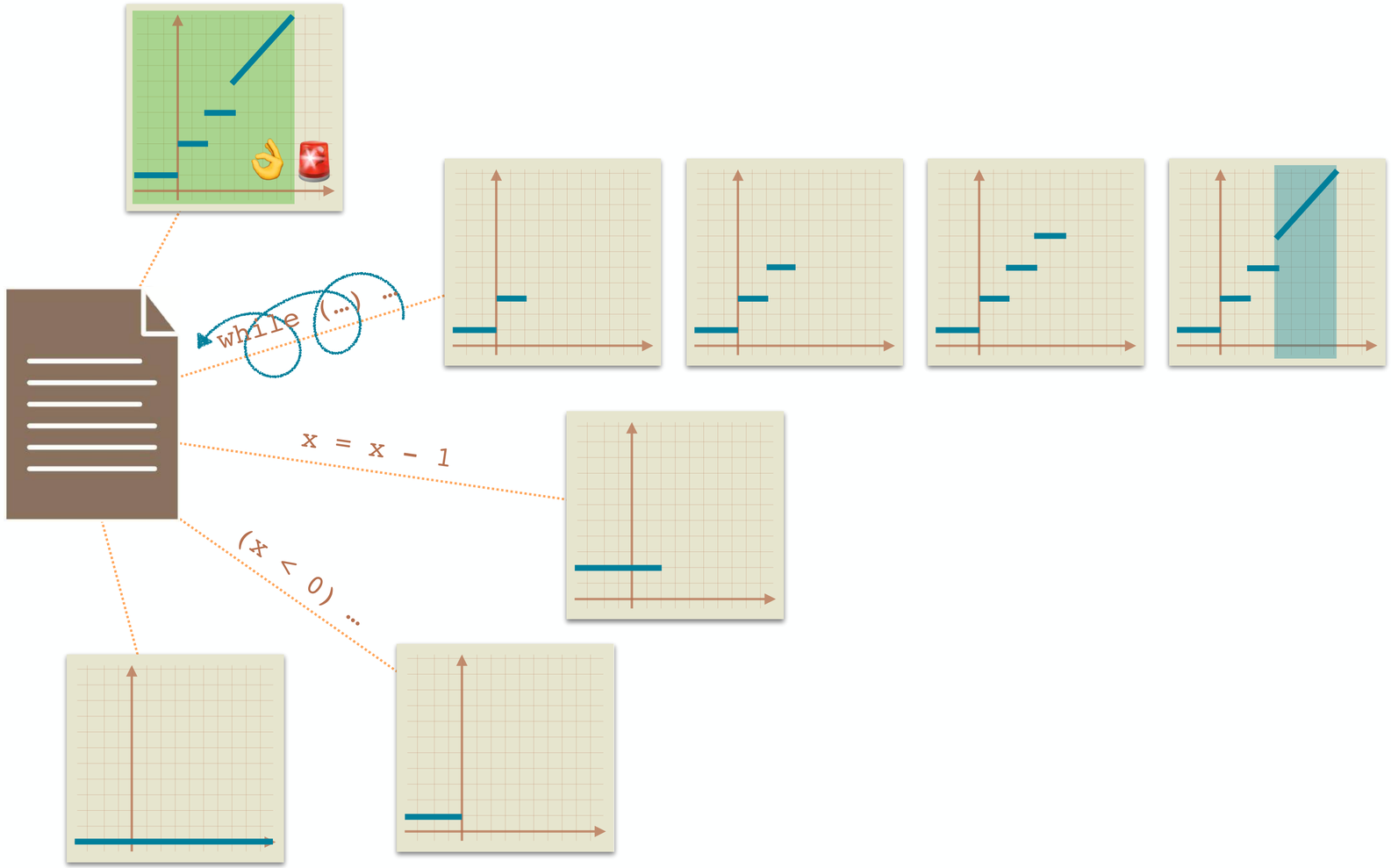








Termination Resilience Static Analysis



Termination Resilience Static Analysis

Approximation Join or Resilience Join?

```
function f(x) {
  1a ← [-∞, +∞]
  2z ← 10
  3if (a*a ≥ 0) then
      while 4(z ≥ 0) do
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  else
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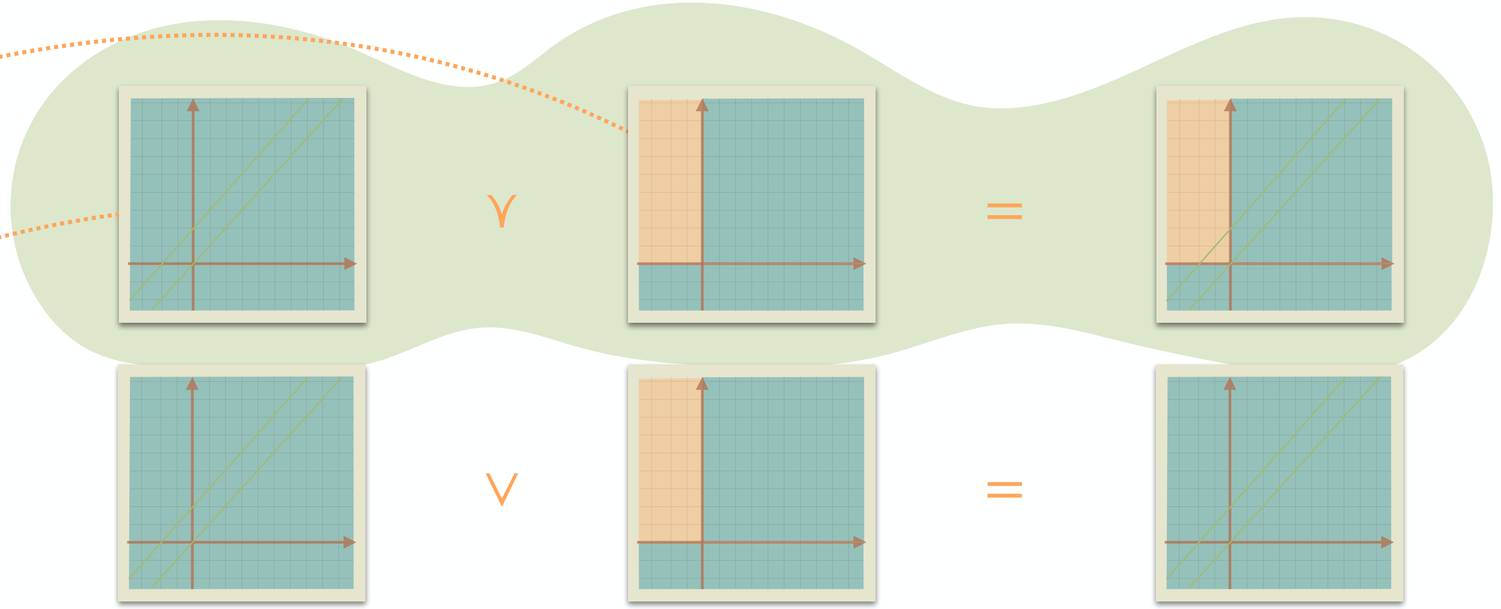
```



Termination Resilience Static Analysis

```
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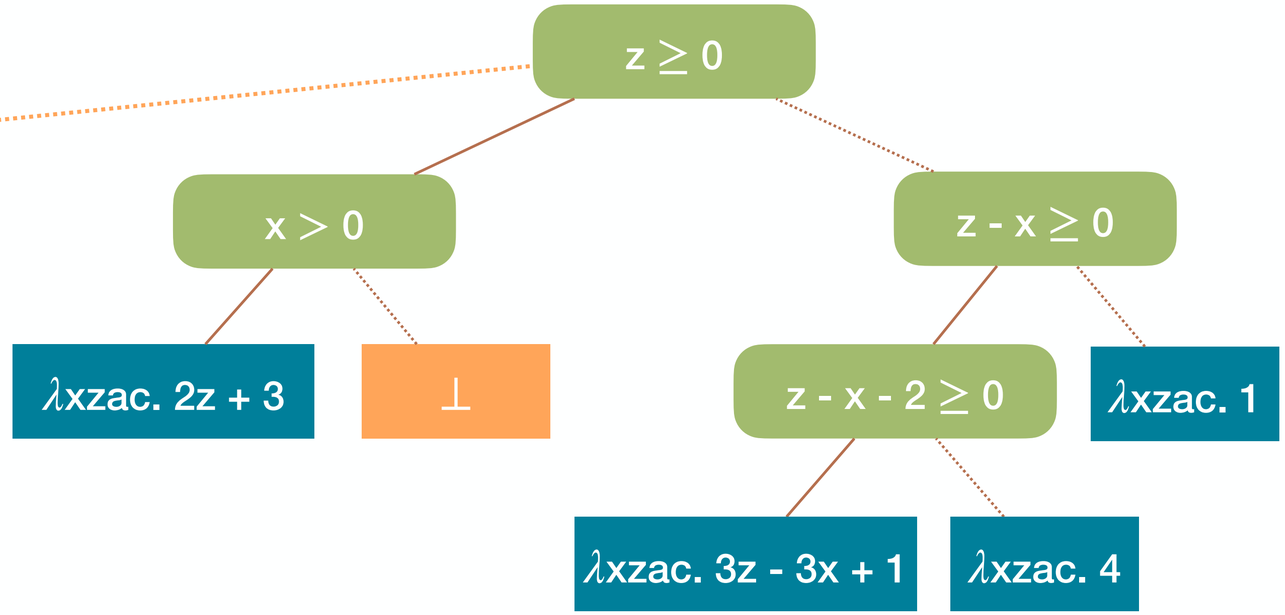
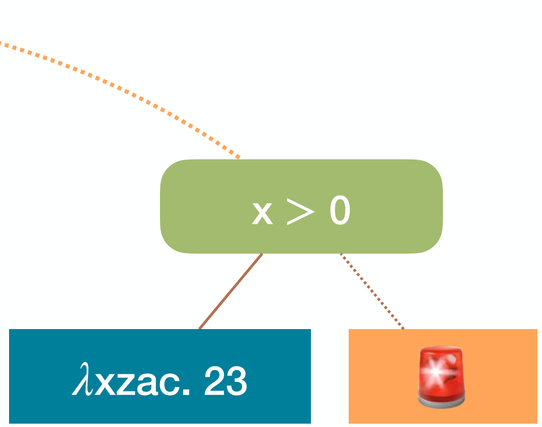


Termination Resilience Static Analysis

```
function f(x) {
  [1]a ← [-∞, +∞]
  [2]z ← 10
  [3]if (a*a ≥ 0) then
      while [4](z ≥ 0) do
        [5]z ← z - x
      od[6]
  else
      while [7](z ≥ x) do
        [8]c ←[-2, 1]
        [9]z ←z + c
      od[10]
  fi

}[11]
```



Termination Resilience Static Analysis

3-Step Recipe

practical tools
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abstract semantics, abstract domains
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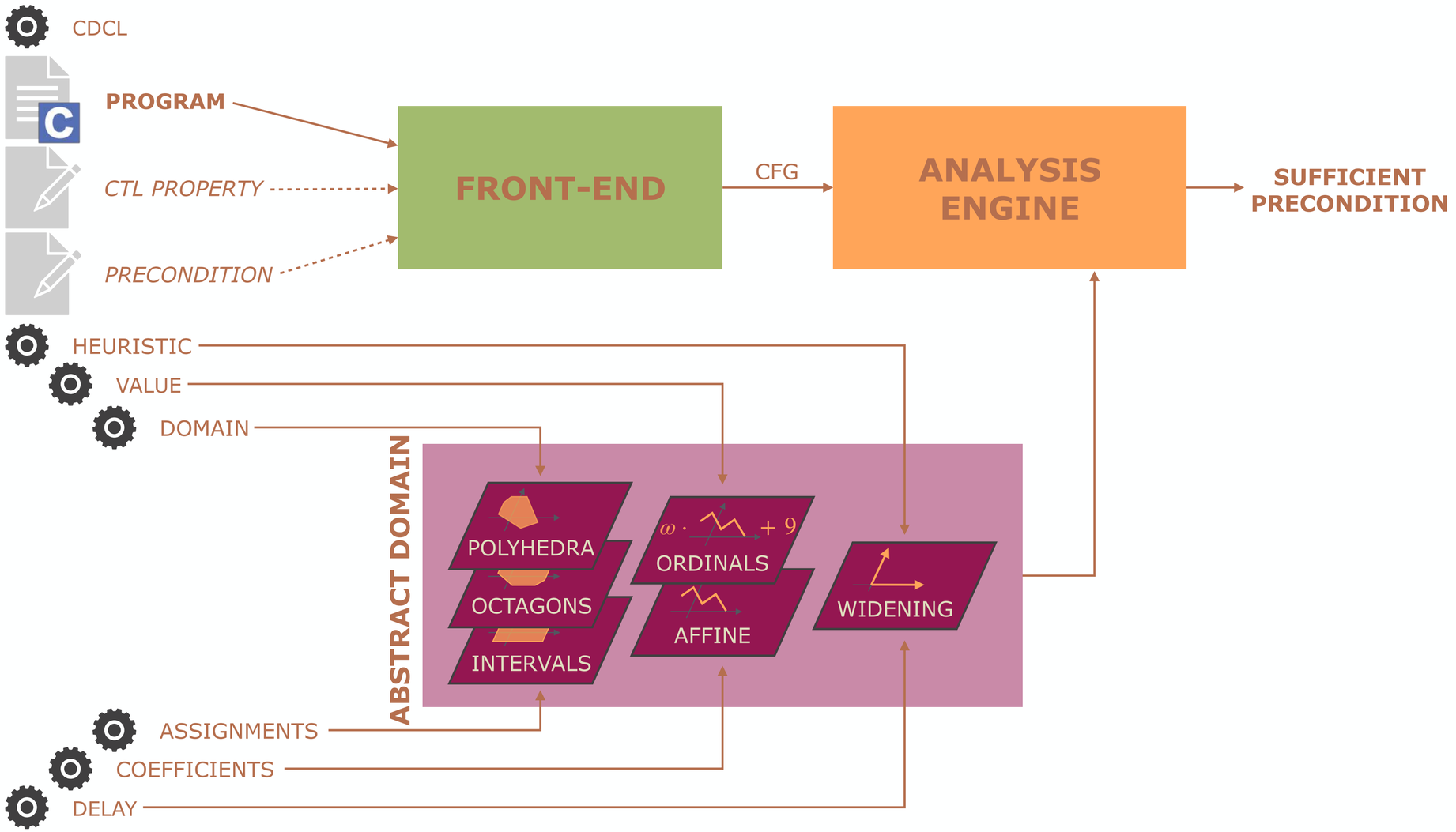
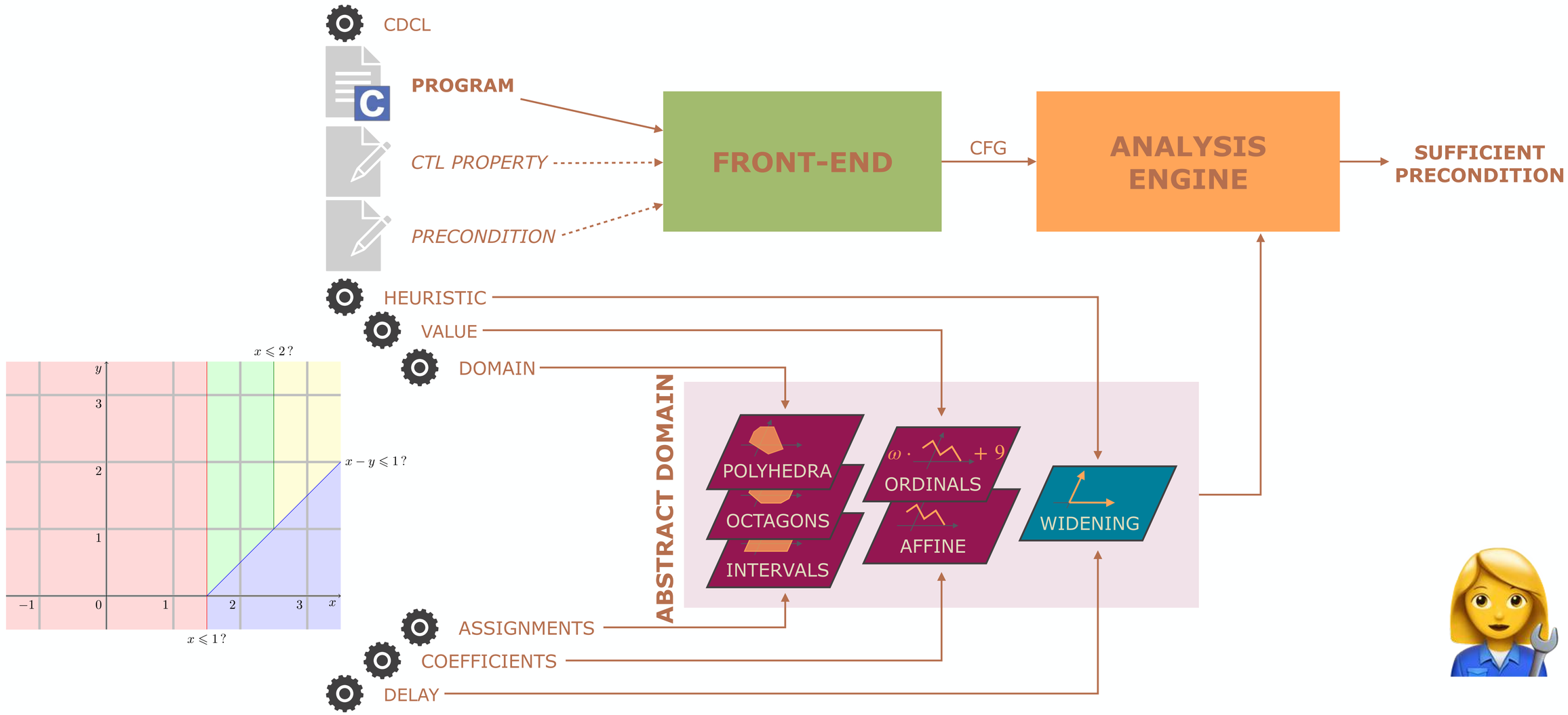
mathematical models of the program behavior

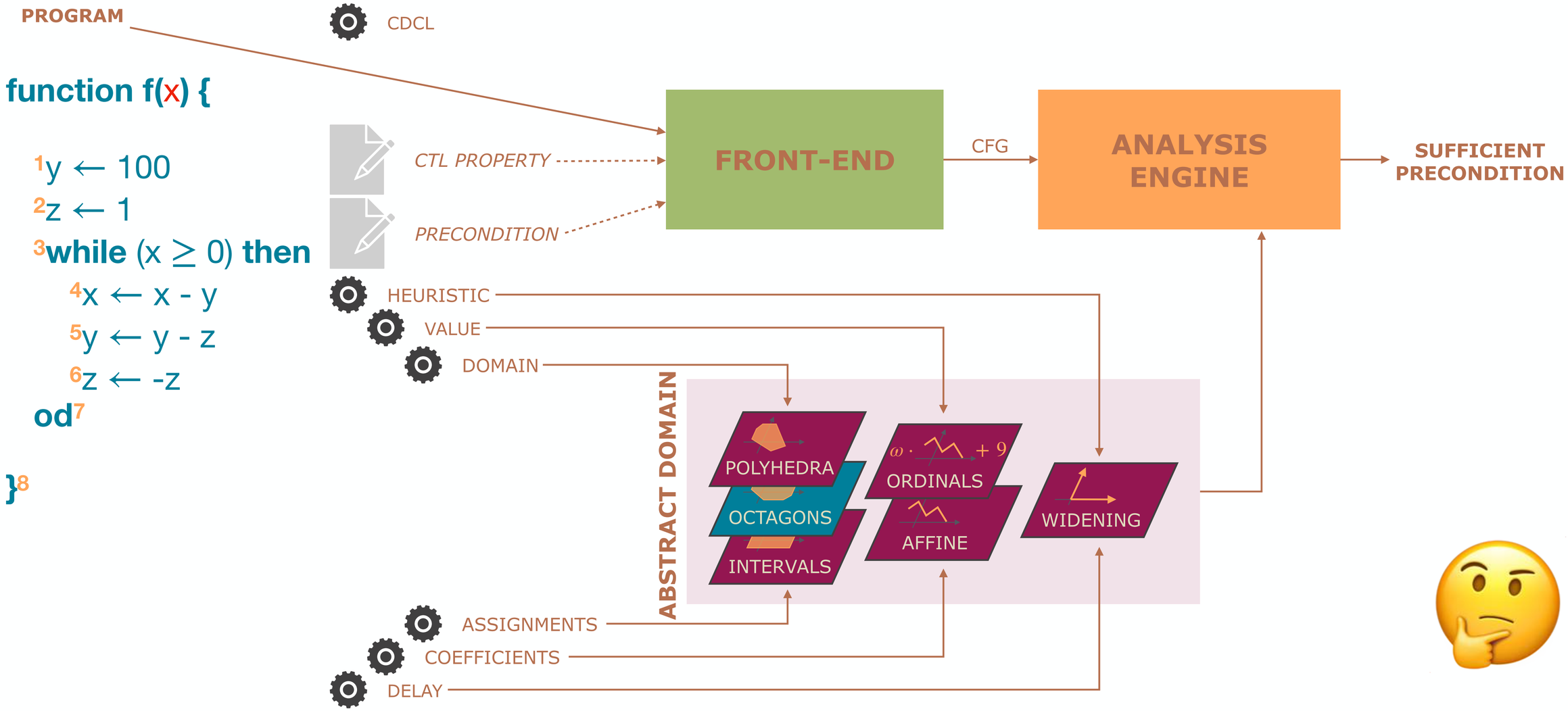
FuncTion


CDCL
PROGRAM
CTL PROPERTY
PRECONDITION
FRONT-END
CFG
ANALYSIS ENGINE
SUFFICIENT PRECONDITION
HEURISTIC
VALUE
DOMAIN
ABSTRACT DOMAIN
POLYHEDRA
OCTAGONS
INTERVALS
ORDINALS
AFFINE
WIDENING
ASSIGNMENTS
COEFFICIENTS
DELAY

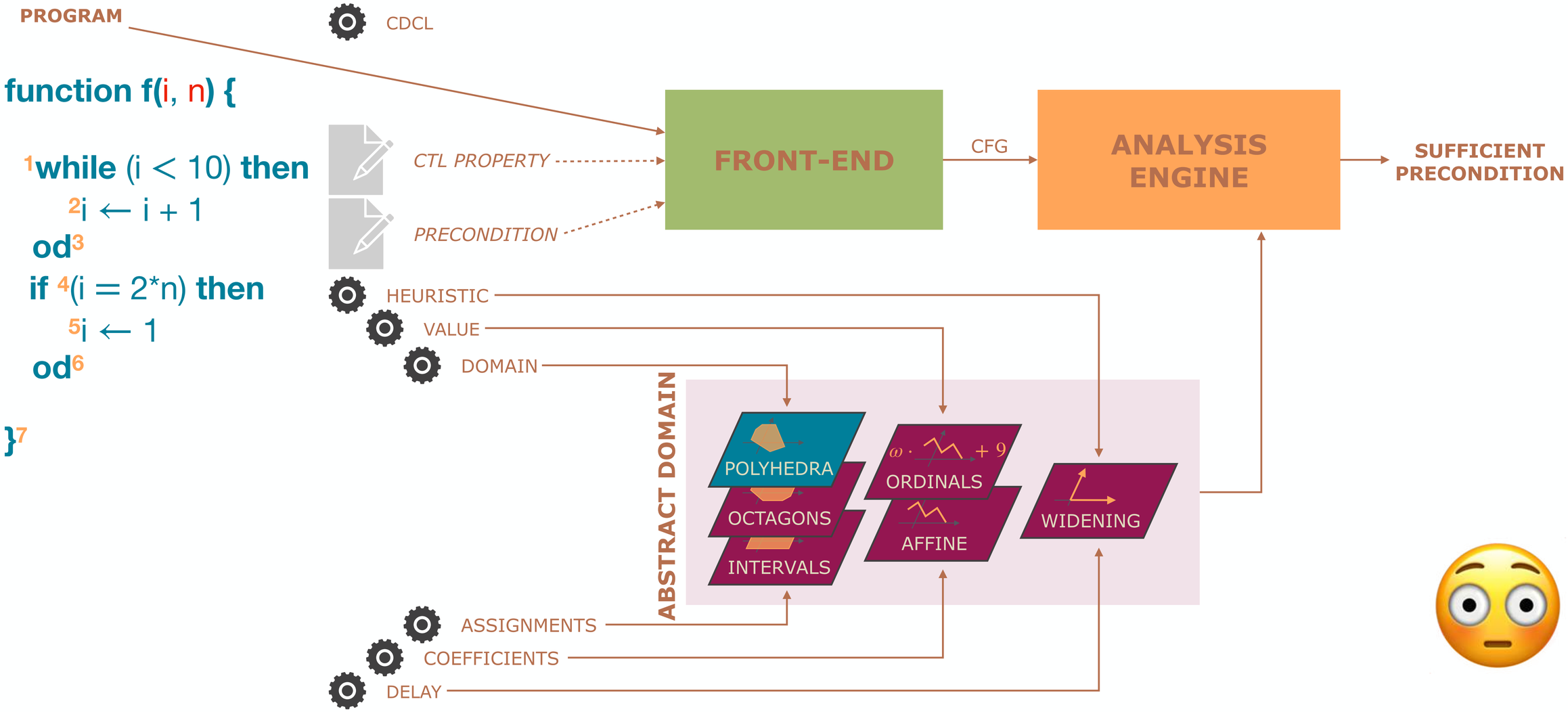
A Silent Bug in the Widening Operator



Non-Terminating (Forward) Analysis

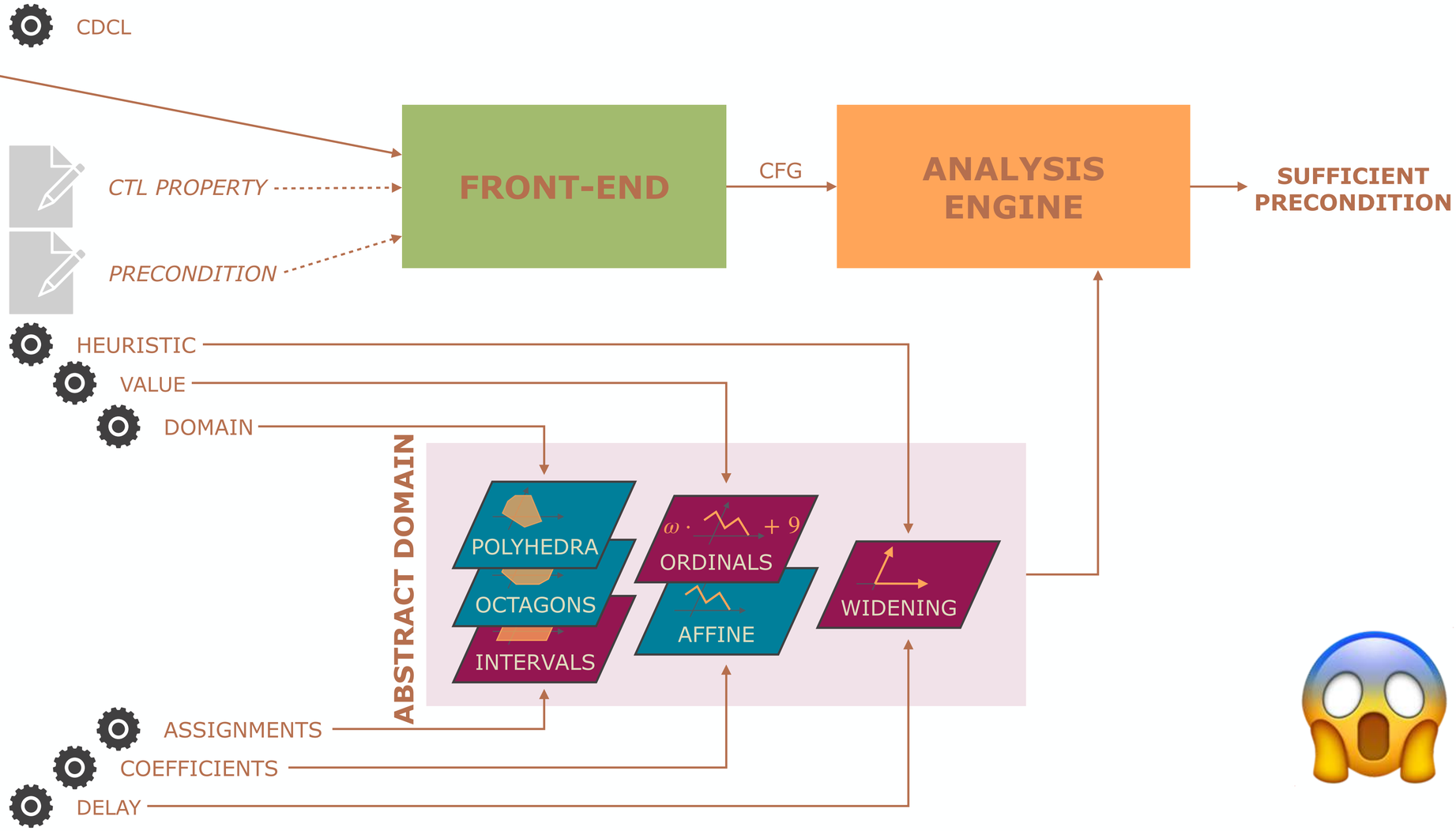


Unexpected Analysis Imprecision #1

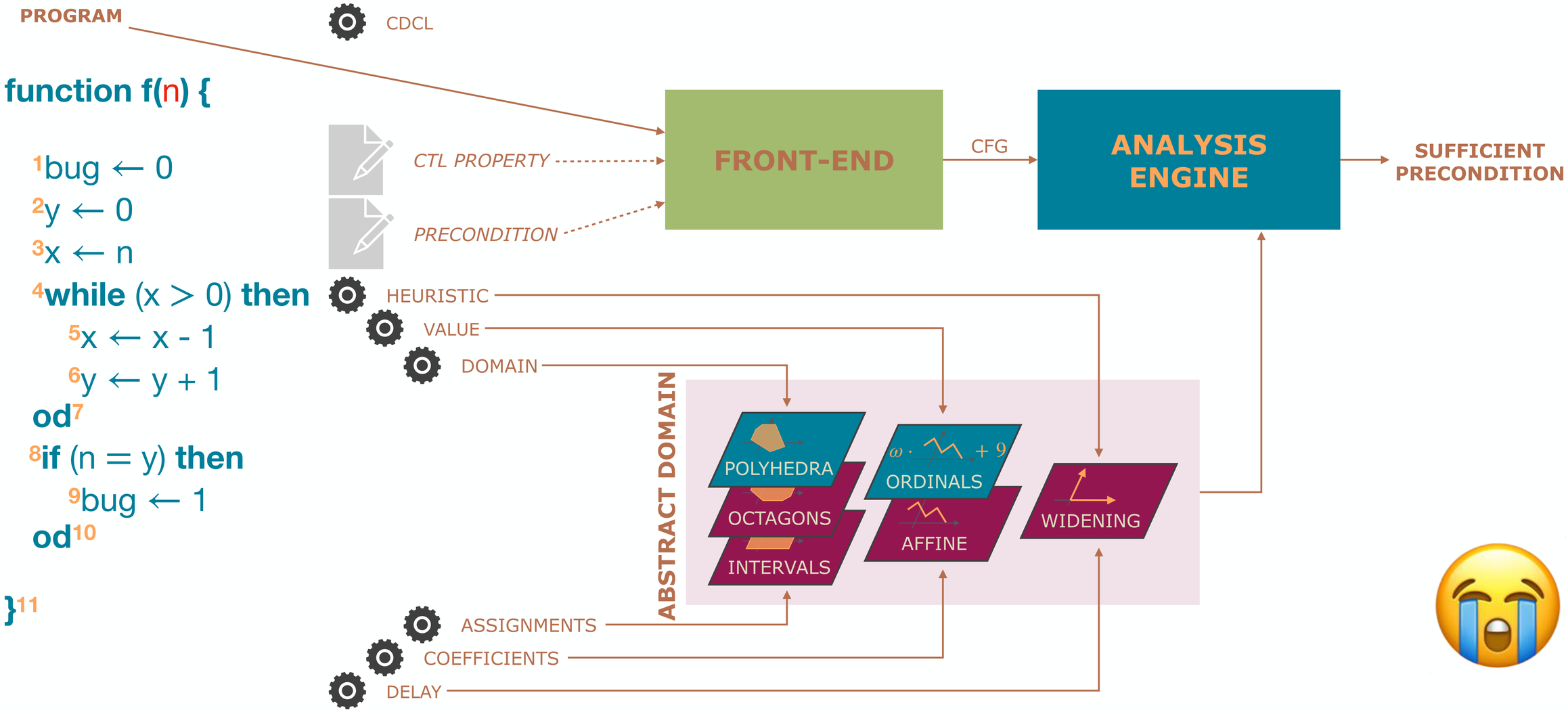


Unexpected Analysis Imprecision #2

```
function f(c) {
  [1]r ← 0
  [2]s ← 4
  [1]while (s > 0) then
      if [4](c ≥ s) then
          [5]c ← c - 1
          [5]r ← r + 1
          [2]s ← s - 1
      else
          [2]s ← s - 1
      endif
   od[3]
}[7]
```



Unexpected Analysis Imprecision #3




HELP

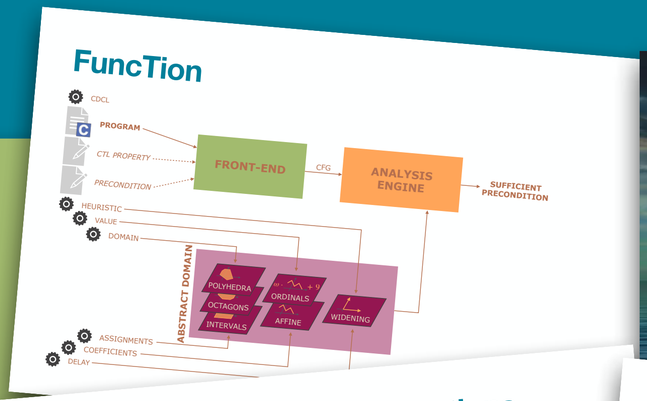
Termination Resilience Static Analysis

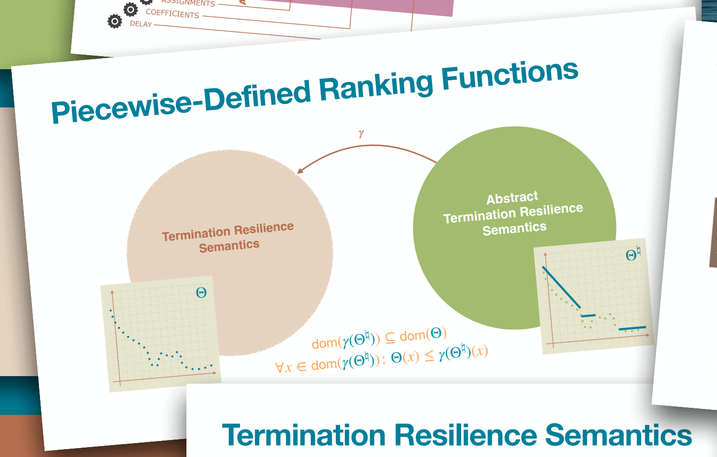
3-Step Recipe

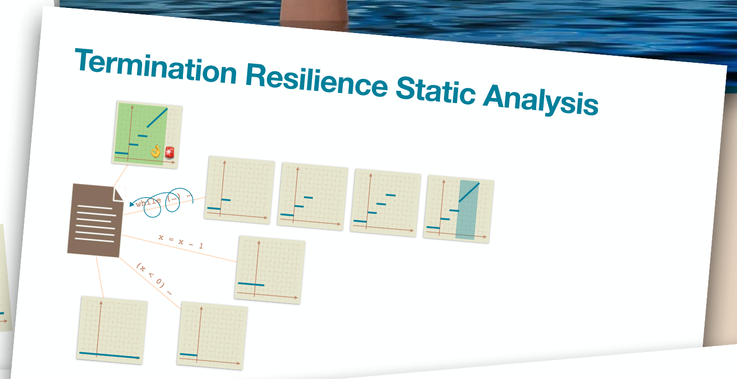
practical tools

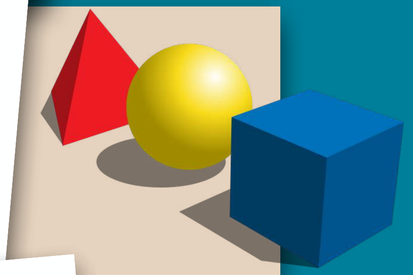
abstract semantics
abstract domains

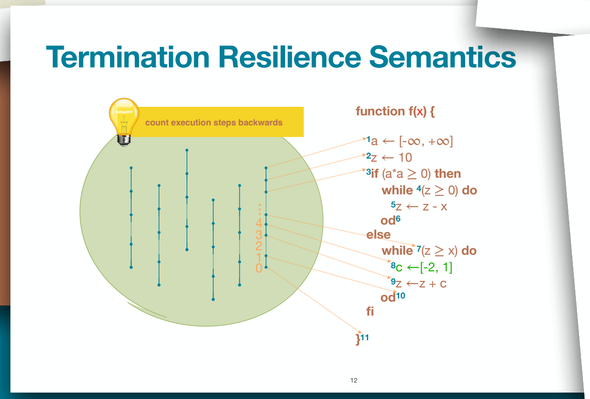
concrete semantics

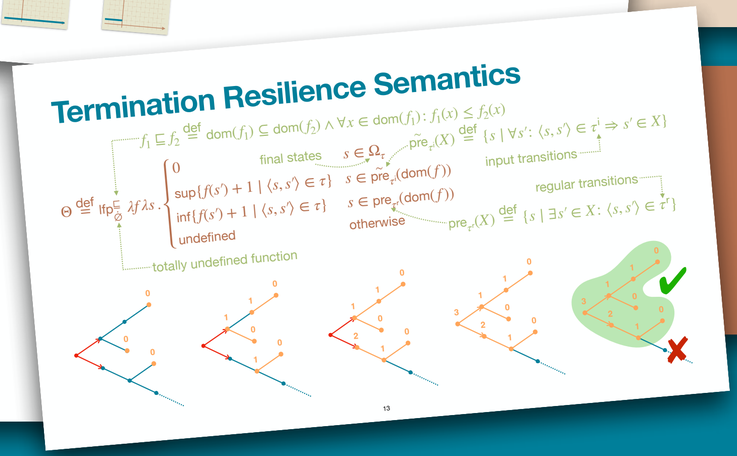












THANKS!